A possible internal space

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Given a crystal structure of space, which is face-centred cubic lattice $\mathcal{A}_1$ of the cell, the cell is a sphere with diameter $l_0 \sim 10^{-18}$m and can be deformed. The perfect $\mathcal{A}_1$ is vacuum space and defected $\mathcal{A}_1$ is matter space. The time is continuity of the physical process. The symmetry group of the vacuum space is the point group O_h and T_d,the symmetry of the matter space is the compact disconnected Lie group $S(O_h) = \{R(\gamma_\alpha^1, \gamma_\alpha^2, \gamma_\alpha^3)T_\alpha, T_\alpha \in O_h\}$ and $S(T_d) = \{R(\gamma_\alpha^1, \gamma_\alpha^2, \gamma_\alpha^3)T_\alpha, T_\alpha \in T_d\}, R(\gamma_\alpha^1, \gamma_\alpha^2, \gamma_\alpha^3) \in SO(3, C)$.The matter field $\Psi(\vec{r}, t)$ is set of all position vectors in matter space-time.The matter field obeys local gauge field invariance $\Psi'(\vec{r}, t) = exp(-i\theta^\alpha(\vec{r}, t)T_\alpha)\Psi(\vec{r}, t)$. T_α are generators, whose algebras $A(O_h)$ and $A(T_d)$ are the group algebra of O_h and T_d,respectively.The $A(O_h)$ and $A(T_d)$ are Lie algebras. $A(O_h) = A(O^+) \oplus A(O^-), A(O^\pm) = \frac{1 \pm \sigma}{2} A(O), A(O) = \sum_{i=1}^5 \oplus A_0^i \oplus A_1 \oplus A_2 \oplus A_2', A(T_d) \cong A(O)$, σ is space inversion operation,A_2 is Lie algebra of SU(3),$A_2'$is another Lie algebra of SU(3), A_1 is Lie algebra of SU(2), A_0 is Lie algebra of $U(1)$.The representations of $A(O_h)$ was that the Gell-Mann's quarks (uds) and their multiplets.The representations of $A(T_d)$ is just the all eighteen leptons $(l_\nu, l, l^c)_L^s$,$s = e, \mu, \tau$,(basis,and conjugate basis) and their multiplets (as same as quark(cbt) multiplets).The interactions are $A(O_h)$ and $A(T_d)$ local gauge field,which are $\prod_{i=1}^5 \otimes U_i(1) \otimes SU_{Spin}(2) \otimes SU_{Flavor}(3) \otimes SU_{Color}(3)$. A pure geometrization gravitational theory are the manifold (M), metric (g_{ab}) and connection (Γ_{bc}^a), gravity action is $S = \int_M d^4x\sqrt{-g}R$, the matter spacetime is manifold M. And the connection is decided completely by the tangent vector bundle structure of the manifold, which is calculated by the translation property of the matter spacetime. Therefore, I have a choice to unify the all present particles and their interactions in the internal space, i.e., the special structure of spacetime.

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CONTENTS

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I. INTRODUCTION

After centuries of scientific exploration, human deeply understand the natural world and their law step by step. In Newton(1642-1727) era, one established the theory in absolute space and absolute time, and founded on the classical mechanics and other classical theories. In this theoretical framework, the existence of matter is independent on the space and time, the interaction between matters also is independent on the space and time. In Albert Einstein(1879-1955) era, critically inherited the scientific achievements of the Newton's era, one set up the relativistic space-time structure [1, 2], and created the quantum theory of particles [3–10].Having the space-time theory and dynamics theory, one study the micro particles from the two aspects: the structures of the particles and the interactions between particles.

Since 1897, J.J.Thomson who found the electron[11] created the first era in micro particle physics. Later, in the experiment which particles pass through the thin foil material E.Rutherford [12, 13] found nuclei and proton p, and then in 1932, J.Chadwick found neutron[14], C.D.Anderson found positron[15] using cloud chamber. It is hoped that the matter world is made up of this a few small "bricks" base. So these particles known as elemental particles. But after 1947, one found a lot of elemental particles which has the basic characteristics of particles as like as e, p, n. One have found more than about 700[16] until 1992.The word *elemental particles* has lost its meaning. People try to classify these particles and find their inner link. Most acknowledged categories show in Table I.

TABLE I. Classify of element particles

Categories	Gauge particles	Leptons	Mesons	Baryons
Examples	$\gamma, W^{\pm}, Z^0$	ν_e, e	π, η	p, n, Λ

The inner link between hadrons(mesons and baryons), S.Sakata [17] consider the internal symmetry SU(3) between the p, n and Λ, but encountered serious difficulties. After that M.Gell-Mann and Y.Neeman introduce the concept of quarks [18–22], which the all hadrons are the complex of quarks. They appointed three quarks (uds) (flavor) as a basic representations of SU(3), and the mesons and baryons are multidimensional irreducible representations of SU(3).Because these multidimensional irreducible representations may be expressed by the direct product of the three dimensional basic representations, the mesons and baryons are the composition with quarks. Later, D.W.Greenberg[23] introduced quark colors to solve the statistical difficulties of baryon structure.

One have presented many kinds of interactions to solve the structure of the material world. More basic interactions include: (1) the gravitational interaction which summed up by Newton as early as the 17th century and geometrized by Einstein;(2) the electromagnetic interaction which been founded by A.M.Ampere, M.Faraday, J.C.Maxwell and others according to unification of the electric and magnetic phenomena in the 19th century;(3) the weak interaction which widely exists between the lepton and lepton, lepton and hadron, hadron and hadron;(4) the strong interaction which is another kind of interaction between hadron and hadron. The ratio of strength of the four interactions is gravity: weak: electricity: strong $\approx 10^{-39}/10^{-12}/10^{-2}/1$.In the understanding process of the interactions, people try to answer how to describe the interaction and these interactions can been or can been not unified.

First, one use *force* to describe interactions, such as universal gravitation, Coulomb force. Later, develop the field, such as the gravitational field, electromagnetic field. Such a description of the interaction does not connected with the symmetry of physical systems. Einstein first connected the gravitational field with the generalized coordinate transformation invariance of space and time, established the general theory of relativity. This is the best gravity theory until now, although its quantization met insurmountable difficulties of negative probability[24].Later, H.Weyl and F.E.London connected the electromagnetic interaction with local phase transformation invariance[25, 26].The crucial significance event is in 1954, Yang C.N and Mills R.L. introduced local gauge invariance thought to strong interaction and got success[27].It is almost confirmed the thought which the gauge field theory is the most effective tool for characterizing the interactions. Later development of GWS[28–31],weak-electromagnetic unification theory, theory of QCD, etc., were further confirmed this idea.

Because the gauge field is inherently linked to the symmetry of physical systems, the symmetry of the system become the main object of study, and the interaction connected with the structure of the particles, and there is possible to unify several kinds of interactions by finding a higher symmetry. Successful example is that S.L.Glashow, S.Weinberg and A.Salam (GWS) theory, which unify the weak interaction and the electromagnetic interaction [28–31].H.George and S.L.Glashow have do beneficial try to unify the strong, the electroweak interaction[32, 33].It is worth pointing out that all these symmetry, in addition to gravitational interaction originated in the space-time symmetry, the others are internal symmetry of the system. As to the precise meaning of *internal*, nobody can exactly formulate. But one thing is common, which is different from the space and time, or outside of the space and time.

To sum up, particle physics includes three aspects: the space-time structure, particle structure and interactions. In Newton's era, the space and time is continuous, absolute, particles do not depend on the space and time structure, the interactions don't depend on the space and time structure and the particles. In Einstein's era, the space and time is continuous, relative, particles do not depend on the space and time structure, but it depends on the symmetry of the internal space, interactions are not dependent on the space and time structure, but they are dependent on the symmetry of internal space, as shown in Table 2.

TABLE II. The relation of spacetime,particles and interactions

Era	Spacetime	Particles	Interactions
Newton	continuous absolute	independed on spacetime and interactions	independed on spacetime and interactions
Einstein	continuous relative	independed on spacetime and interactions	independed on spacetime,but depended on symmetry of internal space

From Table II, the interactions and particles have combined now, even though this combination is not perfect, but the trend has been recognized and become the symmetry of the internal space. And the internal space is independent of spacetime, this shows that the particle and interaction is independent of spacetime structure. Particles are exist in spacetime, however, it appeared the following questions: (1)How to unify the particles granularity and the continuity of spacetime? How to interpret vacuum in spacetime? (2)General relativity tells us that spacetime are geometry, but matter are not geometry, so, how to exist a non-geometric thing (matter) in a geometric spacetime? (3) How to connect the internal space with the spacetime?

To answer the above questions, although there are many tries, but is unable to reconcile in the existing theory. And in terms of the unity of the material world, the most natural and the most general idea is that the spacetime, particles and interactions are interlinked, namely the interactions (including particles structure) result from the spacetime symmetry, the granularity of the particles is derived from the granularity of spacetime - discontinuity (i.e., spacetime quantization).This idea is difficult to achieve in Newton and Einstein spacetime, it requires us to jump out of the Einstein or Newton spacetime and into other kind of spacetime. In this spacetime, interactions (of course including particle structure) completely determined by the spacetime symmetry, the only work is to establish a spacetime structure, the rest is only to reason and deduct.

Very lucky, I finally found such a spacetime: space is divided a face-centered cubic lattice of three dimensional element cell, time is a measure of the process, the space and time become vacuum by spatial translational invariance. Particles are the defects of space cell structure. The electromagnetic, weak and strong interaction comes from local gauge invariance with the point group symmetry of the defective spacetime, and the gravitational interaction comes from the translational symmetry of the defective spacetime. These work have done in this paper, the result is consistent with the existing experimental results.

In this paper, the mathematical tools are used by group theory (including finite group and continuous group), group algebra, Lie algebra, differential geometry, the gauge field theory. There are perfect descriptions in many general books, so don't do the mathematical preparation in this paper.

After establishing spatial structure, this paper gives the concept of matter, vacuum space, matter space, and then, their space structures are studied.Section II studied the symmetry of the space, and got the relation between space symmetry with interaction.Section III researched the concept of time . Section IV specific to construct the present particles.Section V is specific to study spatial symmetry and got electromagnet, weak and strong interaction. Section VI specialize in gravitational interaction. Section VII does a summary for this article. Several conclusions of group algebra are given in Appendix.

II. STRUCTURE OF SPACE AND MATTER

A. Vacumm space and matter space

There are two kinds of space. One kind is a structure that constructed closest by the spherical element cell in the face-centered cubic lattice, which has the highest symmetry and does not flaws. This structure is a vacuum. Because the cell is three-dimensional, so the vacuum is three-dimensional. The size of the cell, namely diameter l_0,is waiting to be determined. Lee T.D. has ever a maximum, $l_0 \sim 10^{-18}$m[34], in Section V of this paper I got $l_0 \sim 0.957 \times 10^{-18}$ m in a case. The cell has other properties, which are sure by compared with the experiment. For example, to make the interaction of particles show in experiments now,the cell must be out of shape, it has detail studies in later sections. Another kind is a flawed face-centered cubic lattice, call it matter space. There are four kinds of matter space as shown in Figure 1.

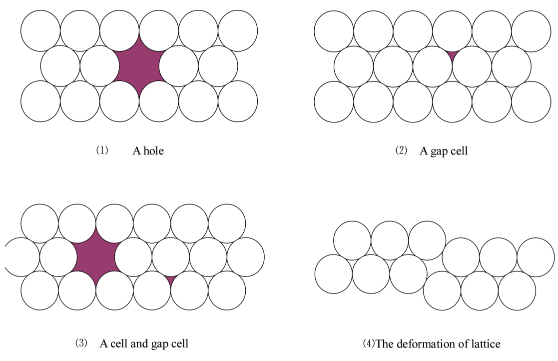

Fig. 1 Four kinds of matter space

(1) Only a hole. Be lack of a cell in face-centered cubic structure. It corresponds to the particle state. (2) Only a cell in the gap of the face-centered cubic, it corresponds to the antiparticles. (3) There are both holes and gap

cell. It corresponds to the particles and anti-particles. (4) No hole and no gap cell, but only deformation of structure. It corresponds to the gauge particles.

Worth pointing out what we can change and observe is in a space between cell and cell, and we don't know what inner of a cell is,also do not need to know. Because all observable phenomena occurred in the nature are in the space between cells, namely the outside of the cell.

In this space, that the vacuum is not empty is well understood. When a cell is stroked from its location into the gap, this cell lattice will become the third matter space, corresponds to the particles and anti-particles pairs.

Such an intuitive space structure, it determines all the particles and their interactions.

B. Space structure

Many celles closely arranged in a straight line, this serial of cell is called the close packing line, as shown in Figure 2.

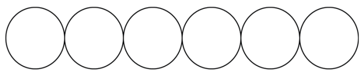

Fig. 2 The close packing line

To arrange many close packing line into a plane with most closely way.The plane is called close packing layer, as shown in Figure 3.This surface is even in the presence of cell center. Here most closely means that the first close packing line set together the adjacent close packing line with translational $\frac{1}{2}l_0$ (l_0 cell diameter).Close packing layer has the following properties.(1) The distance between the two close packing line $D = \frac{\sqrt{3}}{2}l_0 < l_0$;(2) The three cell center of the adjacent cells shall form an equilateral triangle with side length l_0;(3) Every cell contact with 6 adjacent cells, and the close packing layer can be divided with flat hexagonal grid, each of flat hexagonal grid contained in 3 cells and 6 gaps (The party is not filled by cells.).Therefore, every cell in the close packing layer corresponds to two gaps and their ratio is 1:2.

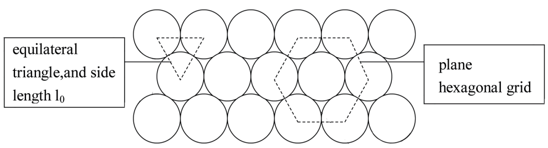

Fig. 3 Close packing layer

As shown in Figure 4,there is only one kind of the most close method to overlay close packing layer B on the close packing layer A,which let cell of layer B is located on gap of layer A. We get close packing double layer, denoted with symbol AB.Because the cell number of the every close packing layer are equal, and the gap of each layer more than twice as many as cell, the cell of layer B can be put on a half of the gap of layer A, the other half of the gap is not covered.This gap,which is covered with four cells and connected the centers of the four adjacent cells, just make a regular tetrahedron. This gap is called tetrahedral gap, denoted by T. Every cell of layer B forms a tetrahedral gap.Similarly, every cell of layer A forms a tetrahedral gap too. So the numbers of tetrahedral gap in double AB is twice the numbers of each layer cell. In another half space of the double AB, the centers of surrounding 6 celles constitute an octahedron, and is called octahedral gap, denoted by O. The number of octahedral gap is equal to the cell number of each layer.

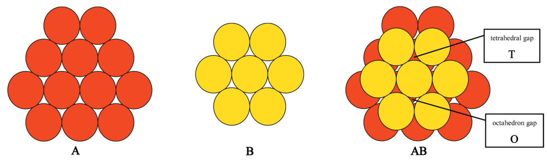

Fig.4 The structure of double layer

There are two kinds of method which let layer C packed most closely on the double AB. One is to let the cell of the layer C in tetrahedral gap of AB. So layer C is equal to layer A. The following pile method is same. It is hexagonal close packed structure, denoted by $\mathcal{A}_3$, as shown in Figure 5.

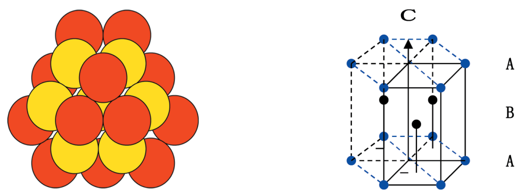

Fig.5 Hexagonal close packed structure A_3

But, there is a very special direction $\vec{c}$ in this kind of close packing,which do not coincide with the isotropic space in the existing experiments. Therefore, the space structure can't be $\mathcal{A}_3$. Another is to let the cell of the layer C locate in octahedron gap of AB. The status of this three layer ABC is inequitable, following pack are just repeat of the three layers ABC. Such close packing is face-centered cubic close packing, denoted $\mathcal{A}_1$, as shown in Figure 6.

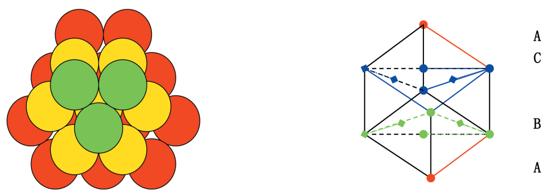

Fig.6 Face-centered cubic A_1

We use the symmetric group to describe the symmetry of space. There are two kind symmetries respectively because there are two kind spaces of vacuum space and matter space.

C. Symmetry of vacuum space

Space defined by Section II(A), perfect $\mathcal{A}_1$ is a vacuum space. By the lattice theory[35], the space group of $\mathcal{A}_1$ is simple space group, its any symmetry operation (that is, the space group elements) can be expressed as the product of translational transformation and point group elements.The translational transformation of $\mathcal{A}_1$ constitutes a full face centered translation group F:

$$T = T_l[E, \Gamma_{0\frac{1}{2}\frac{1}{2}}, \Gamma_{\frac{1}{2}0\frac{1}{2}}, \Gamma_{\frac{1}{2}\frac{1}{2}0}] \tag{1}$$

The T_l is cell translational transformation with integer lattice vector $(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$namely

$$\vec{x} \to \vec{x}' = T_1\vec{x} = \vec{x} + \mathbf{l} = \vec{x} + n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3; \tag{2}$$

where n_1, n_2, n_3 are integer,E is unit element,namely identical transformation $\Gamma_{f_1f_2f_3}$ are translation transformation with fraction vector, namely

$$\begin{aligned}\vec{x} \to \vec{x}' &= \Gamma_{f_1f_2f_3}\vec{x} \\ &= \vec{x} + f_1\mathbf{a}_1 + f_2\mathbf{a}_2 + f_3\mathbf{a}_3; (0 < f_1, f_2, f_3 < 1).\end{aligned} \tag{3}$$

There are two kinds of point group of lattice $\mathcal{A}_1$.One is point group with the center of the cell, and another is point group with the center of the gap.Because there is only one kind of cell, the point group with the center of cell is O_h.

In lattice $\mathcal{A}_1$ there are two kinds of tetrahedral gap T and octahedral gap O. So, the point group with the center of the tetrahedral gap T is tetrahedral full symmetry group T_d; the point group with the center of the octahedral gap O is octahedral full symmetry group O_h.

D. Symmetry of matter space

Defined by Section II(A),defective $\mathcal{A}_1$ is matter space. There are many kinds of matter space.

(1) There is only the hole in lattice.

As shown in Figure 1(1),the position of the each cell around the hole are different.As a result, there are not translational symmetry and the point group symmetry as same as in vacuum space.

(2)There is only gap cell.

As shown in Figure 1(2), the conclusion is equal to only one hole space.Just the point group depend on the tetrahedral gap or octahedral gap.The point group of the tetrahedral gap is the T_d. The point group of the octahedral gap is the O_h.

(3) There is a pair of hole and gap at the same time.

As shown in Figure 1(3), there is not translation invariance and the point group symmetry.In the lattice with a hole, the octahedral gap is symmetrical distribution around the hole, and there is space inversion symmetry with hole center. So the octahedral gap with O_h symmetry. Although the tetrahedral gap around the hole is symmetrical distribution, but there is not with space inversion symmetry.Therefore, tetrahedral gap array with T_d symmetry. This shows that the symmetry of the hole depends on which kind of gap which the cell goes into. If the cell is to enter octahedral gap, the hole has (approximate, namely not to consider cell deformation) O_h symmetry. If the cell is to enter the tetrahedral gap, the hole has T_d symmetry. That is to say, in the matter space with a pair of a hole and gap cell,the point group symmetries of the hole and the gap cell are the same. This is very important. If considering hole corresponds to the particle, gap cell corresponding antiparticle, this ensures the particle and anti-particle come in pairs.

(4) There is only lattice deformation.

As shown in Figure 1(4), all symmetries of space are destroyed.

E. Local gauge invariance of matter space-T_d and O_h local gauge invariance,interactions and particles

There is a hole in the matter space, for example. So, as a result of the existence of the hole, make symmetry of other cell and gap change relative to the vacuum space, and also change the spatial translational invariance. So that, if another hole or gap cell enters the matter space, it feels a different from the vacuum space. I will prove below that these different feelings is the so-called interaction (i.e., we are now found interaction in the experiment).

Let $\Psi\{\vec{r_n}\}$ denote states of a hole or gap cell in matter space (i.e., particles and antiparticles), the $\vec{r_n}$ is the space coordinates for center of number n cell or gap cell, n = 1,2,.... Considering the point group symmetry after a hole or gap cell entering into the vacuum space, operated by the point group O_h or T_d a hole state are

$$\Psi'\{\vec{r_n}\} = \Psi\{T_\alpha\vec{r_n}\}. \tag{4}$$

The $T_\alpha \in O_h$ or T_d. For gap cell has the same results.

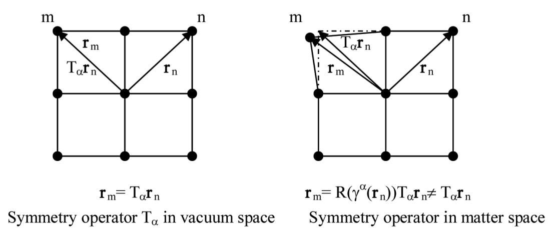

Figure 7 Symmetry operator

For Formula (4), because the each cell of lattice is not distinguishably, there is not physical effect under the operating of $T_\alpha \in O_h$ or T_d,see Fig.7, namely

$$\{\vec{r_n}\} = \{T_\alpha\vec{r_n}\}, (physically) \tag{5}$$

Therefore, the state does not any change after operated by point group in a vacuum space, namely

$$\Psi'\{\vec{r_n}\} = \Psi\{T_\alpha \vec{r_n}\} = \Psi\{\vec{r_n}\}. \tag{6}$$

According to the gauge theory there are not interaction in the vacuum space.

If $\Psi\{\vec{r_n}\}$ is state of a matter space, there is not the symmetry of point group O_h or T_d, namely

$$\{\vec{r}'_n\} = \{T_\alpha \vec{r}_n\} \neq \{\vec{r}_n\}, (T_\alpha \in O_h \text{ or } T_d). \tag{7}$$

If require $\{\vec{r}'_n\} = \{\vec{r}_n\}$the symmetry operations are not $T_\alpha \in (O_h, T_d)$.To make the same, its operation is, for each $\vec{r_n}$, after operated by T_α must to do some adjustments. For example, the turn angle of $\vec{r}'_n$ turned by T_α is too big, $\{\vec{r}'_n\} \neq \{\vec{r}_n\}$. In order to make the $\{\vec{r}'_n\} = \{\vec{r}_n\}$, we need turn back a little angle. See the right part of Figure 7.This is equivalent to

$$R(\gamma^1_\alpha, \gamma^2_\alpha, \gamma^3_\alpha) T_\alpha \vec{r}_n. \tag{8}$$

Where $R(\gamma^1_\alpha, \gamma^2_\alpha, \gamma^3_\alpha) \in SO(3, R); \gamma^i_\alpha (i = 1, 2, 3; \alpha = 0, 1, 2, ., N$,N is the number of point group element) are three parameters of group $SO(3, R)$ for operator T_α.In the rectangular coordinates (x, y, z), they are turning the angle of $\gamma^1_\alpha, \gamma^2_\alpha, \gamma^3_\alpha$ around three axis (x, y, z), and

$$R(\gamma^1_\alpha, \gamma^2_\alpha, \gamma^3_\alpha) = R_z(\gamma^3_\alpha) R_y(\gamma^2_\alpha) R_x(\gamma^1_\alpha). \tag{9}$$

If $|\vec{r}'_n| \neq |\vec{r}_n|$,$\vec{r}'_n$ become larger or less than before $\vec{r}_n$, which can let γ^i_α taking in the complex number:$\gamma^i_\alpha = \xi^i_\alpha + i\eta^i_\alpha, \xi^i_\alpha, \eta^i_\alpha \in R$. Obviously,$\gamma^i_\alpha$ is dependent on $\vec{r}_n$, namely $\gamma^i_\alpha = \gamma^i_\alpha(\vec{r}_n)$.

Pay attention to two points: (1) The so-called distance of $\vec{r}_n$ is the mold of $|\vec{r}_n|$.(2) In the above discussion, we did not consider cell deformation. If consider the deformation of the cell, then l asked us to introduce at least a scalar field $l(\vec{r}_n)$.This is very useful in Section V to construct interaction, $l(\vec{r}_n)$ is the equivalent of the Higgs field[36, 37].

Until now, we analyze the all possible changes of the matter space lattice, found invariance operation of the state $\Psi\{\vec{r}_n\}$ in matter space

$$S_\alpha = R(\gamma^1_\alpha, \gamma^2_\alpha, \gamma^3_\alpha) T_\alpha, \tag{10}$$

where $T_\alpha \in (O_h, T_d)$,$R(\gamma^1_\alpha, \gamma^2_\alpha, \gamma^3_\alpha) \in SO(3, C)$,$\gamma^i_\alpha = \gamma^i_\alpha(\vec{r}_n) \in C, i = 1, 2, 3$.

Firstly we prove that the set $S = \{S_\alpha(\gamma^1_\alpha, \gamma^2_\alpha, \gamma^3_\alpha)\}$ is local Lie group.

(1)S **is group**.

Let $S_\alpha, S_\beta \in S$,

then $S_\alpha S_\beta = R(\gamma^1_\alpha, \gamma^2_\alpha, \gamma^3_\alpha) T_\alpha R(\gamma^1_\beta, \gamma^2_\beta, \gamma^3_\beta) T_\beta$,

because $T^{-1}_\beta R(\gamma^1_\alpha, \gamma^2_\alpha, \gamma^3_\alpha) T_\beta \in SO(3, C)$,$T_\alpha T_\beta \in (O_h, T_d)$,

we have $S_\alpha S_\beta \in S$,which satisfy the closure of multiplcation.It means that the S is a group.

(2) $SO(3, C)$ **is normal subgroup of** S.

Because $T^{-1}_\alpha R(\gamma^1_\alpha, \gamma^2_\alpha, \gamma^3_\alpha) T_\alpha \in SO(3, C)$, $T_\alpha \in (O_h, T_d)$,

we have $T^{-1}_\alpha SO(3, C) T_\alpha = SO(3, C)$.

It means that $SO(3, C)$ is normal subgroup of S.

(3) S **is direct product** $SO(3, C)$ **and** O_h **(or** T_d).

Because of $S = SO(3, C)T (T$ is O_h or $T_d)$,$SO(3, C) \cap T = R(0, 0, 0)E$.Therefore S is direct product $SO(3, C)$ and T. And T is also the normal subgroup of S.

(4) S and $SO(3, C)$ is local isomorphism because the T is discrete subgroup[38].

(5) Because $SO(3, C)$ is a local Lie group,the S is also a local Lie group.

According to a theorem of continuous group[38] the overall properties of S is totally disconnected compact Lie groups. By the theory of Lie groups, the generators of the group S are

$$\left(\frac{\partial S_\alpha \vec{r}_n}{\partial \gamma^i_\alpha}\right)_{\gamma^i_\alpha=0} \frac{\partial}{\partial \vec{r}_n} = \left[\left(\frac{\partial R(\gamma^i_\alpha)}{\partial \gamma^i_\alpha}\right)_{\gamma^i_\alpha=0} \vec{r}_n \frac{\partial}{\partial \vec{r}_n}\right] T_\alpha = (I^i_\alpha)_n T_\alpha, \tag{11}$$

where $i = 1, 2, 3$,n is n^{th} cell, operator

$$(I^i_\alpha)_n = \left[\left(\frac{\partial R(\gamma^i_\alpha)}{\partial \gamma^i_\alpha}\right)_{\gamma^i_\alpha=0} \vec{r}_n \frac{\partial}{\partial \vec{r}_n}\right] \tag{12}$$

is the generator of $SO(3, C)$.

Therefore,the state $\Psi\{\vec{r}_n\}$ becomes $\Psi'\{\vec{r}_n\}$ after operation of the group S,

$$\Psi'\{\vec{r}_n\} = \exp[-i\gamma^i_\alpha (I^i_\alpha)_n T_\alpha] \Psi\{\vec{r}_n\}. \tag{13}$$

Without loss of generality,let $(\vec{r})$ denote $\{\vec{r}_n\}$the Formula (13) becomes

$$\Psi'(\vec{r}) = \exp[-i\Theta^\alpha(\vec{r}) T_\alpha] \Psi(\vec{r}). \tag{14}$$

Where $\Theta^\alpha(\vec{r}) = \gamma^i_\alpha (I^i_\alpha)_n$,$T_\alpha \in T$. The $\Theta^\alpha(\vec{r})$ is an operator,and become the eigenvalue $\theta^\alpha(\vec{r})$ after action of $\Theta^\alpha(\vec{r})$ on the state $\Psi(\vec{r})$.Therefore

$$\Psi'(\vec{r}) = \exp[-i\theta^\alpha(\vec{r}) T_\alpha] \Psi(\vec{r}). \tag{15}$$

In consideration of the matter space evolution, we can lead a parameter t to describe this evolution, and we have following formula for every $(\vec{r}, t)$

$$\Psi'(\vec{r}, t) = \exp[-i\theta^\alpha(\vec{r}, t) T_\alpha] \Psi(\vec{r}, t). \tag{16}$$

In next section we shall point out that the parameter t is time. The Formula (16) means that the state is invariance after this operating by S. According to the gauge field theory we know as soon as that the Formula (16) is the local gauge invariance field with the generators T_α.

About this gauge invariance theory the next work is only to determine the algebra of T_α.From the Table 14 in Appendix,for any $T_\alpha, T_\beta \in T$,we have

$$[T_\alpha, T_\beta] = C_{\alpha\beta\gamma} T_\gamma; T_\alpha, T_\beta, T_\gamma \in T. \tag{17}$$

Where $[T_\alpha, T_\beta] \equiv T_\alpha T_\beta - T_\beta T_\alpha$,$C_{\alpha\beta\gamma}$ are the structure constants which determined completely by the group O_h

(or T_d).The Formula (17) means that the group algebra of O_h (or T_d) is Lie algebra. The rest of question is that what kind algebra is the group algebra of O_h or T_d. Very lucky,the group algebra of group O can be decomposed with the direct sum of following semi-simple Lie algebras:

$$A(O) = \sum_{i=1}^{5} \oplus A_0^i \oplus A_1 \oplus A_2 \oplus A_2^{'}. \tag{18}$$

In similar way

$$A(T_d) = \sum_{i=1}^{5} \oplus A_0^{id} \oplus A_1^d \oplus A_2^d \oplus A_2^{'d} \cong A(O). \tag{19}$$

$$A(O_h) = (O^+) \oplus A(O^-), A(O^{\pm}) = \frac{1 \pm \sigma}{2} A(O) \cong A(O). \tag{20}$$

This suggests the isomorphism of T_d group algebra and O group algebra. The O group algebra can be decomposed into the direct sum of five A_0 Lie algebra (corresponding to $U(1)$ group), one A_1 Lie algebra (corresponding to $SU(2)$ group) and two different A_2 Lie algebra (corresponding to two different $SU(3)$ group).The group algebra of O_h can be decomposed into direct sum of two group algebra, $A(O^+)$ and $A(O^-)$, which is isomorphic to the O group algebra $A(O)$. The T_d group algebra $A(T_d)$ is isomorphic to the O group algebra but is not the same. Details see Appendix.

In opinion of the Yang-Mills local gauge field theory, this is equivalent to say: for O_h gauge field, it is equivalent to the $\prod_{i=1}^{5} \otimes U_i(1) \otimes SU_{spin}(2) \otimes SU_{Flavor}(3) \otimes SU_{Color}(3)$ group of inner space (add space inversion symmetry), T_d gauge field is equivalent to the $\prod_{i=1}^{5} \otimes U_i(1) \otimes SU_{spin}(2) \otimes SU_{Flavor}(3) \otimes SU_{Color}(3)$ group of internal symmetry (without space inversion symmetry).If the $SU(3)$ corresponds to the electroweak interaction, $SU(3')$ corresponding to the strong interaction, $SU(2)$ corresponds to the spin interaction, so we can describe these interactions in a unified framework (O_h or T_d gauge field),that is to say, I unified the strong and the weak, electromagnet interactions (see Section V).The particles are the representations of group algebra $A(O_h)$ and $A(T_d)$,respectively.If the $SU(2)$ corresponds to spin, $SU(3)$ corresponds to the flavor, the $SU(3')$ corresponds to the color, we can construct all particles!(see Section IV).After the three sections detailed study, the fact as expected. So, I have linked closely interactions (including gravitational interaction) and particles and space-time structure! The following works from Formula (16), the first is to construct the representations of the group algebra, and then one to introduce the interaction depended on Formula (16) and to compare with the experiment. Before these work we need to establish the concept of time.

III. TIME

Description of continuity of the physical process is the time. Compare continuity of two physical process, that is to compare the time of the two physical processes, you must first select a relatively standard process, to compare the continuity of the two physical processes respectively with the continuity of the standard process, so as to achieve the aim of comparing two physical process continuity.

A. Motion and reference frame

The vacuum lattice is the undeformed face centered cubic lattice $\mathcal{A}_1$.Therefore, all cells are equivalent. If a hole or a gap cell enters the vacuum (here 'enter' can be interpreted as: the matter space with a hole or gap cell is equivalent to the vacuum after the hole or gap cell entering in).There is still the spatial translational invariance and point group symmetry with center of the hole or the gap cell. This indicates existence of movement: keep to the point group invariance and translation invariance with the center (i.e.,O_h or T_d), I call this kind of motion as the uniform motion (or static).The state with a hole or gap cell (corresponding to a particle) is called a state of uniform motion (or stationary state).In a vacuum, therefore, the hole or gap cell which is in a state of uniform motion don't feel any changes, it is not able to use some kind of their own different states to describe the process of their self (i.e., the different state sequence changing).Thus, a single hole or gap cell itself can't describe the process itself.

If a hole or gap cell enter the matter space, the translational invariance and the point group invariance is no longer found, namely the hole or gap cell will feel a different environment after the translation or rotations. As a result, a hole or gap cell can be to determine whether or not they are motion in matter space by determining the environment changes (i.e., motions have definition).So, the matter space provides a reference frame for a hole or gap cell to describe itself motion. Note that there are not any special requirements of the matter space, it means that every matter space can be as a reference frame, i.e., the reference frame is matter space. In this reference frame, we can introduce a set of coordinates for a hole or gap cell in which there is the concept of position and direction. By measuring the change of position and direction we can determine whether motion or not.

Now let's look at two holes or gap cells A and B into the matter space S. At this moment, A hole or gap cell A is in the matter space S with B denoted $S_B = S + B$. Similarly, B is in $S_A = S + A$, it is clear that these two matter space S_A and S_B are different, the description of A and B are in different reference frame.If the effect of A and B for matter space S is not too big, then $S_A \approx S_B \approx S$, which we can use a unified reference frame S to describe the motion of A and B.

Due to a hole or gap cell into the matter space (that is, in a certain reference frame) there is no longer a vacuum translational invariance and point group invariance, so the motion of holes or gap cells will no longer is a state of uniform motion (or static) in principle. If after a hole or gap cell entering the matter space the translation invariance and point group invariance hold, so a hole or gap cell motion is a uniform motion (or static), when the matter space is equivalent to establish a set of coordinates in a vacuum. We call this matter space as inertial system. It is obvious that the above approach is only an approximation. So, there is no strict inertial system.

B. Definition of time

With a reference frame, you can describe motion of a hole or gap cell. So there will be a motion sequence of a hole or gap cell - process. This tells us forward how to describe and compare the two processes. In the reference frame S,motion sequence - the process can be describe with trajectory, and the trajectory in S is a set of position which a hole or gap cell go through.

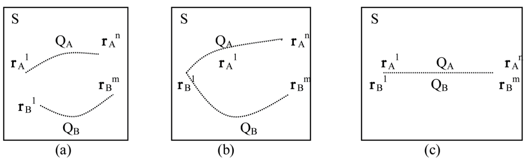

Fig. 8 Relation of two trajectories

For example, denote the trajectory of a hole or gap cell A as $Q_A = \vec{r}^i_A, i = 1, 2, ..., n$.In a similar, denote the trajectory of a hole or gap cell B as $Q_B = \vec{r}^i_B, i = 1, 2, ..., m$,the relations of Q_A and Q_B shown in Fig.8.(a) $Q_A \cap Q_B = \emptyset$;(b) $Q_A \cap Q_B \neq \emptyset$,but $Q_A \neq Q_B$,means intersection;(c) $Q_A \cap Q_B = Q_A$ for $m \geq n$, $Q_A \cap Q_B = Q_B$ for $n \geq m$.

Here we can see only two points: (1) trajectory is the same ;(2) whether meet together. Such as the situation (a) never met together, (b) may meet, but not sure. We call the meet at the simultaneous.For any trajectory $Q = \{\vec{r}^i, i = 1, 2, ..., n\}$, we can always do a one-to-one continuous mapping $t : Q \to T \subset R$. T must be a closed set of R. So $t(\vec{r}^i) = t^i \in T \subset R$, that is equivalent to appoint a parameter t^i at any space point $\vec{r}^i$ in the trajectory Q,$\vec{r}^i$ can be denoted by $\vec{r}^i(t^i)$.Due to the requirement for the mapping t is one-to-one continuous mapping; there is a lot of this mapping. For different trajectories Q_A and Q_B, we can introduce two map t_A and t_B respectively to satisfy the above conditions. So that each point on the trajectory Q_A can be denoted by $\vec{r}^i_A(t^i_A)$, each point on the trajectory Q_B can be denoted by $\vec{r}^i_B(t^i_B)$, where t^i_A,$t^i_B \in R$. Now that the t^i_A and t^i_B are the numbers in R, we can compare their size. Because the map t_A and t_B, however, there are uncertainties, therefore we cannot do this kind of comparison before selected mapping t_A and t_B. Therefore, if you want to compare the size of t^i_A and t^i_B, we must make sure of the map t_A and t_B first. If you want to compare the any above two processes, the easiest way is to determine the method which has uniqueness mapping t. Such the mapping t is called time,and the $\vec{r}^i(t^i)$ called event. If two events $\vec{r}^i_A(t^i)$ and $\vec{r}^i_B(t^i)$ are equal in the time, namely $t^i_A = t^i_B$,it is called at the same time. Note here the same time depends on the choice of mapping t,but the definition of the meet is not dependent on the choice of mapping t. Also, we can define the average velocity $\bar{\vec{V}}$ and instantaneous velocity $\vec{V}$,

$$\bar{\vec{V}}(\triangle\vec{r}^i(t^i)) \equiv \frac{\triangle\vec{r}^i}{\triangle t^i} = \frac{\vec{r}^{i+j} - \vec{r}^i}{t^{i+j} - t^i}, \tag{21}$$

$$\vec{V}(\vec{r}^i(t^i)) = \frac{\vec{r}^{i+1} - \vec{r}^i}{t^{i+1} - t^i}. \tag{22}$$

It is well known that there are the lattice vibration wave in the lattice structure matter (which are light and sound waves for atomic crystal), and the lattice wave velocity is completely determined by the lattice structure[39].Because the vacuum and matter space are the lattice structure of cell, the most natural idea is that such lattice can produce and propagate lattice wave, and the lattice wave velocity is determined only by the vacuum or matter space structure. Here the wave velocity is defined as Formula (22).

Let the matter space is the reference frame. In the general reference frame, because the translation invariance and point group symmetry (O_h or T_d) do not set up, where the matter structure is uneven. Therefore the wave velocity which depends on the structure of the matter space is not uniform. However, when the matter space is inertial system (see definition earlier), symmetry is still found. In the inertial system, therefore, the cell lattice wave velocity is isotropy and is constant value everywhere (elected a time definition).Therefore, a process that the lattice wave propagate through cell provides us with a standard process which is consistent with selected inertial system. Other process time can be determined by comparing with the time of the standard process.

The lattice wave must be rectilinear propagation in inertial system space. The trajectory $Q_p = \{\vec{r}^i_p, i = 0, 1, 2, ..., n\}$,where $\vec{r}^i_p$ is positions which are the center coordinates of the cell that the lattice wave in turn go through. And $\vec{r}^i_p$ satisfy the equation,

$$\vec{r}^j - \vec{r}^i \propto \vec{r}^k - \vec{r}^i, \tag{23}$$

for any $k > i, j > i$.We select a nature mapping t,which satisfy the equation,

$$t^i_p \equiv t^i(\vec{r}_p) = t^i_0. \tag{24}$$

Where t_0 is a number depended on the inertial system. According to the velocity definition Formula(21) and (22),

$$\bar{\vec{V}} = \vec{V} = \frac{\vec{r}^{i+1} - \vec{r}^{i}}{t^{i+1} - t^{i}} = \frac{l_0}{t_0}\hat{e}. \qquad (25)$$

Where $\hat{e}$ is the unit vector which in direction of lattice wave propagation.The l_0 is the cell diameter in the inertial system.Let

$$\frac{l_0}{t_0} = c. \qquad (26)$$

The c is the lattice velocity. From Formula (26)the constant t_0 is the time that the lattice wave goes through one cell in the inertial system.

In the inertial system S, we chose the propagation of the lattice wave as standard process, and will choose t_0 as the unit of time. To select a method which other process can compare with the standard process. Set process track of any hole or gap cell A, $Q_A = \vec{r}^{i}_A, i = 1, 2, ..., n$.In $Q_A$any two adjacent $\vec{r}^{i+1}_A$ and $\vec{r}^{i}_A$ can determine a direction $\hat{e}$, namely $\vec{r}^{i+1}_A - \vec{r}^{i}_A = l_0\hat{e}$.Assume there is a lattice wave propagating along the direction $\hat{e}$,and then meet with A in $\vec{r}^{i}_A$ position.We can always find M position in direction $\hat{e}$, departing from $\vec{r}^{i}_A$ with m lattices, the lattice wave is reflected back at M and again meet A at $\vec{r}^{i+1}_A$.Choose map t_A of process Q_A,

$$\triangle t^{i}_A = t^{i+1}_A - t^{i}_A = (2m-1)t_0, \qquad (27)$$

t^{i}_A is time. It means that the time what A goes through this one cell is equal to the time interval what the lattice wave meet A twice. From Formula(27), for different hole or gap cell processes with m can be different,it means the velocity,

$$\vec{V} = \frac{l_0}{(2m-1)t_0}\hat{e}, \qquad (28)$$

is different. In this way we get unique time for every position in Q_A.

From the above consideration, the general reference frame (the matter space) is impossible to find time definition suitable for the whole space. Therefore,the time only defined in local space.

Does the physical cell lattice wave exist?Now experiments show that the light waves can be seen as a kind of cell lattice wave.

In the inertial system, because of the translation invariance, inertial motion requires the trajectory $Q = \{\vec{r}^{i}, i = 1, 2, ..., n\}$ keep translation invariant. This is equivalent to require the difference between any two adjacent cell position in Q are equal,namely

$$\vec{r}^{i+j} - \vec{r}^{i} \propto \vec{r}^{j+1} - \vec{r}^{j}, \qquad (29)$$

for any $\vec{r}^{i+j}, \vec{r}^{i}, \vec{r}^{j+1}, \vec{r}^{j}$.According to the definition of time (Formula(27)) and velocity (Formula(22)),the inertia motion in Formula (29) is uniform motion in a straight line or static (for static,$Q = \{\vec{r}^{i} | \vec{r}^{i} = \vec{r}^{j}$,for any i,j$\}$).

That is to say:

(1) In the inertial system, there are two kinds of equivalent motion state: A uniform motion and stillness for a hole or gap cell. Suggesting that the inertial system S making A uniform motion and the inertial system S' making A static are equivalent, that is equal rights of different inertial system.

(2) Since each inertial system same as the vacuum space structure and the lattice wave velocity is depended on the structure of space, the lattice wave velocity in each of the inertial system should be the same. Therefore, cell lattice wave velocity in different inertial system is equal to c.

The first conclusion is the principle of relativity; the second is the principle of constancy of light velocity. Therefore, in our definition of space and time, Einstein's special relativity remains valid. This suggests that our concept of time and space is reasonable.

Be worth to say that: (1) There is *ether* theory in history. It said that the light is wave of etheric medium. Where the matter and etheric are two kinds of different material. So in the inertial system the light wave velocity that depended on the etheric structure is that the etheric medium is static, it must be change with different ether velocity in different inertial system of relative motion. So the speed of light constant denied the existence of ether [40].And in my theory, lattice wave velocity only depends on the inertial system space structure, the wave velocity of different inertial system are the same because the structure is the same. (2) For two different inertial S and S',can be defined respectively cell diameter l_0 and l'_0, time t_0 and t'_0. Because of lattice wave velocity is constant,

$$\frac{l_0}{t_0} = \frac{l'_0}{t'_0} = c. \qquad (30)$$

This suggests that allows the length of the unit l_0 and time t_0 have different values in different inertial system. But direct numerical comparison in the two inertial system is meaningless, only to return to the same coordinate system.

IV. PARTICLES

Because of matter spacetime symmetry, the particles have Formula (16) in the form of local gauge invariance. So the particles should be representations of generators T_α.There are two kinds of algebra of the generators T_α.One is group algebra $A(O_h)$ of O_h and another is group algebra $A(T_d)$ of T_d.A single hole point group symmetry is O_h or T_d, tetrahedral gap cell symmetry for T_d, octahedral hole symmetry for O_h.

A. Particles of $A(O_h)$:Quark(uds) and their multiplets

1. Representations of group algebra $A(O)$

By Appendix, group O includes 24 T_α, its group algebra denoted by $A(O)$, can be decomposed into direct sum of five A_0 Lie algebra, one A_1 and two different A_2 Lie algebra, namely

$$A(O) = \sum_{i=1}^{5} \oplus A_0^i \oplus A_1 \oplus A_2 \oplus A_2^{'}. \tag{31}$$

Where A_n is algebra of n rank Lie group $SU(n+1)$,A_0^i(i=1,2,3,4,5)denote 5 difference A_0,$A_2^{'}$ denote another A_2.The standard basis of algebra $A(O)$ are :

(1) Five generators of A_0, denoted by X_i, i=1,2,3,4,5;which are the sum of element in the same class.

$$\begin{cases} X_1 = T_0 = E, \\ X_2 = T_1 + T_2 + T_3, \\ X_3 = T_4 + T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} + T_{11}, \\ X_4 = T_{12} + T_{13} + T_{14} + T_{15} + T_{16} + T_{17}, \\ X_5 = T_{18} + T_{19} + T_{20} + T_{21} + T_{22} + T_{23}. \end{cases} \tag{32}$$

(2) Three generators $A, E_\pm$ of A_1 are

$$\begin{cases} A = i\frac{\sqrt{3}}{24}(T_4 + T_5 + T_6 + T_7 - T_8 - T_9 - T_{10} - T_{11}), \\ E_\pm = \frac{\sqrt{3}}{4}[2(T_{12} + T_{17} + T_{20} + T_{23}) \\ \quad - (T_{13} + T_{16} + T_{18} + T_{21}) \\ \quad - (T_{14} + T_{175} + T_{19} + T_{22})] \\ \quad \pm i\frac{3}{4}[(T_{13} + T_{16} + T_{18} + T_{21}) \\ \quad - (T_{14} + T_{15} + T_{19} + T_{22})]. \end{cases} \tag{33}$$

(3)Eight generators $H_1, H_2, E_{\pm\alpha}, E_{\pm\beta}, E_{\pm(\alpha+\beta)}$ of A_2 are

$$\begin{cases} H_1 = \frac{1}{8\sqrt{3}}\{T_1 - T_2 \\ \quad + \frac{1}{2}[(T_{12} + T_{17} - T_{20} - T_{23}) \\ \quad - (T_{14} + T_{15} - T_{19} - T_{22})]\}, \\ H_2 = \frac{1}{32}\{2T_2 - T_1 - T_3 \\ \quad + \frac{1}{2}[2(T_{13} + T_{16} - T_{18} - T_{21}) \\ \quad - (T_{12} + T_{17} - T_{20} - T_{23}) \\ \quad - (T_{14} + T_{15} - T_{19} - T_{22})]\}, \\ E_\alpha = + \frac{1}{8\sqrt{6}}[(T_4 + T_5 - T_6 - T_7) \\ \quad + (T_{12} - T_{17} - T_{20} + T_{23})], \\ E_{-\alpha} = + \frac{1}{8\sqrt{6}}[(T_8 + T_9 - T_{10} - T_{11}) \\ \quad + (T_{12} - T_{17} + T_{20} - T_{23})], \\ E_\beta = - \frac{1}{8\sqrt{6}}[(T_4 - T_5 - T_6 + T_7) \\ \quad - (T_{14} - T_{15} - T_{19} + T_{22})], \\ E_{-\beta} = - \frac{1}{8\sqrt{6}}[(T_8 - T_9 - T_{10} + T_{11}) \\ \quad - (T_{14} - T_{15} + T_{19} - T_{22})], \\ E_{+(\alpha+\beta)} = + \frac{1}{8\sqrt{6}}[(T_4 - T_5 + T_6 - T_7) \\ \quad + (T_{13} - T_{16} - T_{18} + T_{21})], \\ E_{-(\alpha+\beta)} = + \frac{1}{8\sqrt{6}}[(T_8 - T_9 + T_{10} - T_{11}) \\ \quad + (T_{13} - T_{16} + T_{18} - T_{21})]. \end{cases} \tag{34}$$

(4)Another eight generators

$H_1^{'}, H_2^{'}, E_{\pm\alpha}^{'}, E_{\pm\beta}^{'}, E_{\pm(\alpha+\beta)}^{'}$ of $A_2^{'}$ are

$$\begin{cases} H_1^{'} = & \frac{1}{8\sqrt{3}}\{T_1 - T_2 \\ & - \frac{1}{2}[(T_{12} + T_{17} - T_{20} - T_{23}) \\ & - (T_{14} + T_{15} - T_{19} - T_{22})]\}, \\ H_2^{'} = & \frac{1}{32}\{2T_2 - T_1 - T_3 \\ & - \frac{1}{2}[2(T_{13} + T_{16} - T_{18} - T_{21}) \\ & - (T_{12} + T_{17} - T_{20} - T_{23}) \\ & - (T_{14} + T_{15} - T_{19} - T_{22})]\}, \\ E_{\alpha}^{'} = & \frac{1}{8\sqrt{6}}[(T_4 + T_5 - T_6 - T_7) \\ & - (T_{12} - T_{17} - T_{20} + T_{23})], \\ E_{-\alpha}^{'} = & \frac{1}{8\sqrt{6}}[(T_8 + T_9 - T_{10} - T_{11}) \\ & - (T_{12} - T_{17} + T_{20} - T_{23})], \\ E_{\beta}^{'} = & - \frac{1}{8\sqrt{6}}[(T_4 - T_5 - T_6 + T_7) \\ & + (T_{14} - T_{15} - T_{19} + T_{22})], \\ E_{-\beta}^{'} = & - \frac{1}{8\sqrt{6}}[(T_8 - T_9 - T_{10} + T_{11}) \\ & + (T_{14} - T_{15} + T_{19} - T_{22})], \\ E_{+(\alpha+\beta)}^{'} = & \frac{1}{8\sqrt{6}}[(T_4 - T_5 + T_6 - T_7) \\ & - (T_{13} - T_{16} - T_{18} + T_{21})], \\ E_{-(\alpha+\beta)}^{'} = & \frac{1}{8\sqrt{6}}[(T_8 - T_9 + T_{10} - T_{11}) \\ & - (T_{13} - T_{16} + T_{18} - T_{21})]. \end{cases} \tag{35}$$

A particle should be the representation of the 24 generators $X_i, A_1, A_2, A_2^{'}$ together.We discuss respectively below.

(1) The representations of the A_1. A_1 is Lie algebra of Lie group $SU(2)$,rank 1.There is one quantum number S, we call it spin, the irreducible representation as Table III, this suggests that the particles should be half integer spin Dirac particles.

TABLE III. Spin representation

Class	Representation(S)	Dimension
0	0	1
1	$\frac{1}{2}, -\frac{1}{2}$	2
...	...	...
l	$\frac{l}{2}, \frac{l-1}{2}, ..., -\frac{l-1}{2}, -\frac{l}{2}$	$l+1$

(2)The representations of the A_2.A_2 is Lie algebra of Lie group $SU(3)$,rank 2. There is two quantum number Y, I_3.The basic representation denoted by (uds) with dimension 3.The irreducible representation as Table IV.

This is the same as Ge11-manns three quarks theory. Other multiple representations are exactly equivalent Ge11-manns quark theory, not repeat here.

TABLE IV. Quantum number of (uds)

	Y	I_3
u	$\frac{1}{3}$	$\frac{1}{2}$
d	$\frac{1}{3}$	$-\frac{1}{2}$
s	$-\frac{2}{3}$	0

(3)The representations of the A_2'. A_2' is Lie algebra of Lie group $SU(3)$,rank 2. There is two quantum number Y^c, I_3^c.The basic representation denoted by (RBG) with dimension 3.The irreducible representation as Table V.

TABLE V. Quantum number of (RBG)

	Y^c	I_3^c
R	$\frac{1}{3}$	$\frac{1}{2}$
B	$\frac{1}{3}$	$-\frac{1}{2}$
G	$-\frac{2}{3}$	0

This is the same as Ge11-manns three colors theory. Other multiple representations are exactly equivalent Ge11-manns quark theory, not repeat here[40–42].

(4)The representations of the five A_0. Besides $T_0 = E$ is unit element, the generator number of the other four algebras are only one, it may correspond to the discrete and overall quantum number, such as parity[43] and the total spin S, charge Baryon number, etc.

According to discussion in the above four parts, the representation of a group algebra $A(O)$ - particles with spin S, flavor (uds), color (RBG) and overall quantum number, etc., but the symmetry of actual hole state is higher than group algebra $A(O)$, meet the group of algebra $A(O_h)$.

2. *Representations of group algebra $A(O_h)$*

As can be seen from the Appendix(Section VIII),the group algebra of group O_h can be decomposed into direct sum of two group algebras:

$$A(O_h) = A(O^+) \oplus A(O^-), \tag{36}$$

where

$$\begin{aligned} O^+ &= \{T_i P^+, T_i \in O\}, \\ O^- &= \{T_i P^-, T_i \in O\}, \\ P^\pm &= \tfrac{1}{2}(1 \pm \sigma), \end{aligned} \tag{37}$$

where σ is space inversal.Because $(P^\pm)^2 = P^\pm$,so $P^\pm$ is chirality projection operator. In the Dirac representation chirality projection operator can be take as

$$P^\pm = \frac{1 \pm \gamma_5}{2}. \tag{38}$$

Where γ_5 is Dirac matrix.From Formula (37),$A(O^+)$ and $A(O^-)$ are isomophic to $A(O)$.In this way Formula (36) shows that the representations of group algebra $A(O_h)$ divided into two kinds: one kind is the representation of group algebra $A(O^+)$,

$$\Psi^+ = \frac{1+\gamma_5}{2}\Psi \equiv \Psi_L, \tag{39}$$

called left-handed state, in addition to the factor $\frac{1}{2}(1+\gamma_5)$,the Ψ_L is same as the representation of group algebra $A(O)$.Another kind is the representation of group algebra $A(O^-)$,

$$\Psi^- = \frac{1-\gamma_5}{2}\Psi \equiv \Psi_R, \tag{40}$$

called right-handed state, in addition to the factor $\frac{1}{2}(1-\gamma_5)$,the Ψ_R same as the representation of group algebra $A(O)$.

Here we completely constructed by all the particles made of quarks (uds) from the package representation of group algebra $A(O_h)$.Because the theory consistency of the existing theory of Gell-mann three quarks, so it is completely consistent with the existing experiment.

B. Particles of $A(T_d)$:Leptons and quark(cbt) multiplets

In the Appendix, group algebra $A(T_d)$ and $A(O)$ are isomorphism, also can be decomposed into direct sum of five A_0 Lie algebra, one A_1 and two different A_2 Lie algebra.

(1)Five generator of A_0, denoted by X_i^d, i=1,2,3,4,5, which are the sum of element in the same class.

$$\begin{cases} X_1^d = T_0 = E, \\ X_2^d = T_1 + T_2 + T_3, \\ X_3^d = T_4 + T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} + T_{11}, \\ X_4^d = (T_{12} + T_{13} + T_{14} + T_{15} + T_{16} + T_{17})\sigma, \\ X_5^d = (T_{18} + T_{19} + T_{20} + T_{21} + T_{22} + T_{23})\sigma. \end{cases} \tag{41}$$

(2)Three generators $A^d, E_\pm^d$ of A_1^d are

$$\begin{cases} A^d = i\frac{\sqrt{3}}{24}(T_4 + T_5 + T_6 + T_7 - T_8 - T_9 - T_{10} - T_{11}), \\ E_\pm^d = \frac{\sqrt{3}}{4}[2(T_{12} + T_{17} + T_{20} + T_{23}) \\ \quad - (T_{13} + T_{16} + T_{18} + T_{21}) \\ \quad - (T_{14} + T_{175} + T_{19} + T_{22})]\sigma \\ \quad \pm i\frac{3}{4}[(T_{13} + T_{16} + T_{18} + T_{21}) \\ \quad - (T_{14} + T_{15} + T_{19} + T_{22})]\sigma. \end{cases} \tag{42}$$

(3)Eight generators $H_1^d, H_2^d, E_{\pm\alpha}^d, E_{\pm\beta}^d, E_{\pm(\alpha+\beta)}^d$ of A_2^d are

$$\begin{cases} H_1^d = \frac{1}{8\sqrt{3}}\{T_1 - T_2 \\ \quad + \frac{1}{2}\sigma[(T_{12} + T_{17} - T_{20} - T_{23}) \\ \quad - (T_{14} + T_{15} - T_{19} - T_{22})]\}, \\ H_2^d = \frac{1}{32}\{2T_2 - T_1 - T_3 \\ \quad + \frac{1}{2}\sigma[2(T_{13} + T_{16} - T_{18} - T_{21}) \\ \quad - (T_{12} + T_{17} - T_{20} - T_{23}) \\ \quad - (T_{14} + T_{15} - T_{19} - T_{22})]\}, \\ E_\alpha^d = +\frac{1}{8\sqrt{6}}[(T_4 + T_5 - T_6 - T_7) \\ \quad + \sigma(T_{12} - T_{17} - T_{20} + T_{23})], \\ E_{-\alpha}^d = +\frac{1}{8\sqrt{6}}[(T_8 + T_9 - T_{10} - T_{11}) \\ \quad + \sigma(T_{12} - T_{17} + T_{20} - T_{23})], \\ E_\beta^d = -\frac{1}{8\sqrt{6}}[(T_4 - T_5 - T_6 + T_7) \\ \quad - \sigma(T_{14} - T_{15} - T_{19} + T_{22})], \\ E_{-\beta}^d = -\frac{1}{8\sqrt{6}}[(T_8 - T_9 - T_{10} + T_{11}) \\ \quad - \sigma(T_{14} - T_{15} + T_{19} - T_{22})], \\ E_{+(\alpha+\beta)}^d = +\frac{1}{8\sqrt{6}}[(T_4 - T_5 + T_6 - T_7) \\ \quad + \sigma(T_{13} - T_{16} - T_{18} + T_{21})], \\ E_{-(\alpha+\beta)}^d = +\frac{1}{8\sqrt{6}}[(T_8 - T_9 + T_{10} - T_{11}) \\ \quad + \sigma(T_{13} - T_{16} + T_{18} - T_{21})]. \end{cases} \tag{43}$$

(4)Another eight generators

$H_1^{d'}, H_2^{d'}, E_{\pm\alpha}^{d'}, E_{\pm\beta}^{d'}, E_{\pm(\alpha+\beta)}^{d'}$ of $A_2^{d'}$ are

$$\left\{\begin{aligned} H_1^{d'} =& \frac{1}{8\sqrt{3}}\{T_1 - T_2 \\ & - \frac{1}{2}\sigma[(T_{12}+T_{17}-T_{20}-T_{23}) \\ & - (T_{14}+T_{15}-T_{19}-T_{22})]\}, \\ H_2^{d'} =& \frac{1}{32}\{2T_2 - T_1 - T_3 \\ & - \frac{1}{2}\sigma[2(T_{13}+T_{16}-T_{18}-T_{21}) \\ & - (T_{12}+T_{17}-T_{20}-T_{23}) \\ & - (T_{14}+T_{15}-T_{19}-T_{22})]\}, \\ E_{\alpha}^{d'} =& \frac{1}{8\sqrt{6}}[(T_4+T_5-T_6-T_7) \\ & - \sigma(T_{12}-T_{17}-T_{20}+T_{23})], \\ E_{-\alpha}^{d'} =& \frac{1}{8\sqrt{6}}[(T_8+T_9-T_{10}-T_{11}) \\ & - \sigma(T_{12}-T_{17}+T_{20}-T_{23})], \\ E_{\beta}^{d'} =& -\frac{1}{8\sqrt{6}}[(T_4-T_5-T_6+T_7) \\ & + \sigma(T_{14}-T_{15}-T_{19}+T_{22})], \\ E_{-\beta}^{d'} =& -\frac{1}{8\sqrt{6}}[(T_8-T_9-T_{10}+T_{11}) \\ & + \sigma(T_{14}-T_{15}+T_{19}-T_{22})], \\ E_{+(\alpha+\beta)}^{d'} =& \frac{1}{8\sqrt{6}}[(T_4-T_5+T_6-T_7) \\ & - \sigma(T_{13}-T_{16}-T_{18}+T_{21})], \\ E_{-(\alpha+\beta)}^{d'} =& \frac{1}{8\sqrt{6}}[(T_8-T_9+T_{10}-T_{11}) \\ & - \sigma(T_{13}-T_{16}+T_{18}-T_{21})]. \end{aligned}\right. \tag{44}$$

Comparing Formula (41)-(44) and Formula (32)-(35), although their algebraic structure are the same, but the physical meaning is different.So T_d is another set of generator, their representations is another one which is different from group algebra $A(O_h)$.It means that the particle of $A(T_d)$ is different to the quark particles (uds) of $A(O_h)$.We will structure the particles of $A(T_d)$ in turn below.

1. *The basic representations of group algebra $A(T_d)$*

(1) The representations of the A_1^d. Comparison Formula (42) with Formula (33),we have

$$A^d = A, E_\pm^d = \sigma E_\pm \tag{45}$$

In O group algebra, the $(A, E_\pm)$ constitute a Lie algebra A_1, their representation can be written as $|jm>$, there are

$$\left\{\begin{array}{l} A|jm>= m|jm>, \\ E_\pm|jm>= \sqrt{(j\mp m)(j\pm m+1)}|jm> . \end{array}\right. \tag{46}$$

Now, $(A^d, E_\pm^d)$ also constitute a Lie algebra A_1,their representation can be written as $|jm>^d$,there are

$$\left\{\begin{array}{l} A^d|jm>^d= m|jm>^d, \\ E_\pm^d|jm>^d= \sqrt{(j\mp m)(j\pm m+1)}|jm>^d . \end{array}\right. \tag{47}$$

Using Formula (45) in Formula (47),we have

$$\left\{\begin{array}{l} A|jm>^d= m|jm>^d, \\ E_\pm\sigma|jm>^d= \sqrt{(j\mp m)(j\pm m+1)}|jm>^d . \end{array}\right. \tag{48}$$

So,we have

$$\sigma|jm>^d= |jm>^d . \tag{49}$$

The $|jm>^d$ restricted by Formula (49),we have

$$\frac{1-\sigma}{2}|jm>^d= 0 \tag{50}$$

By definition of Formula (39) and (40), it is same that the particle must be left-handed.This property leads to that the basic representation of group algebra T_d has a left-handed structure. It is the cause of the asymmetry of left and right for the particles.

(2)The representations of the A_2.A_2 is Lie algebra of Lie group $SU(3)$,rank 2. There is two new quantum number Y^d, I_3^d.The basic representation do not be denoted by (uds) and denoted by $(l\nu, l, l^c)_L$ with dimension 3.The irreducible representation as Table VI.

TABLE VI. Quantum number of $(l_{\nu L}, l_L, l_L^c)$)

	Y^d	I_3^d	Q
$l_{\nu L}$	$\frac{1}{3}$	$\frac{1}{2}$	0
l_L	$\frac{1}{3}$	$-\frac{1}{2}$	-1
l_L^c	$-\frac{2}{3}$	0	+1

In Table VI,in addition to the last column Q, the rest of the figures are same as in Table IV. it is required by the Lie algebra A_2. Q meets here

$$Q = I_3^d - \frac{3}{2}Y^d. \tag{51}$$

If the Q as charge quantum number, so they are that $l_{\nu L}$ correspond to left-hand neutrinos,l_L correspond to left-hand leptons, l_L^c correspond to left-handed anti-lepton, and the conjugate representation of A_2 in Table VI are that right-hand anti-neutrinos(i.e.,the anti-particle of the left-hand neutrinos), left-hand anti-leptons(i.e., the anti-particle of left-handed lepton), right-handed leptons (i.e.,the anti-particle of the left-hand anti-leptons).Considering:

$$Y^d = \frac{2}{\sqrt{3}}H_1^d; I_3^d = H_2^d. \tag{52}$$

The Q determined by Formula (41) is eigen operator of Lie algebra A_2 with basis representation $(l\nu, l, l^c)_L$. Such choice of Q as same as Gell-mann-Nishijima relation is meaningful.

(3)The representations of the $A_2^{d'}$. $A_2^{d'}$ is Lie algebra of Lie group $SU(3)$,rank 2. Their basis representations which are different to the basis of A_2,denoted (e,μ,τ).There is two new quantum number $(Y^c)^d$,$(I_3^c)^d$.Their corresponding relation shows in Table VII.

TABLE VII. Quantum number of $(e\mu\tau)$

	$(Y^c)^d$	$(I_3^c)^d$
e	$\frac{1}{3}$	$\frac{1}{2}$
μ	$\frac{1}{3}$	$-\frac{1}{2}$
τ	$-\frac{2}{3}$	0

Therefore, in combination with Table VI and spin state, there are 9+9 lepton states.The one-to-one relation of basis and leptons shows in the Table VIII and Table IX, just the same.

TABLE VIII. The one-to-one relation of basis and leptons

Basis of $A(T_d)$	$l_{\nu L}^e$	l_L^e	$(l^c)^e$	$l_{\nu L}^\mu$	l_L^μ	$(l^c)^\mu$	$l_{\nu L}^\tau$	l_L^τ	$(l^c)^\tau$
leptons	ν_{eL}	e_L	e_L^+	$\nu_{\mu L}$	μ_L	μ_L^+	$\nu_{\tau L}$	τ_L	τ_L^+

TABLE IX. Conjugate representation basis of $A(T_d)$

Conjugate *Basis* of $A(T_d)$	$\overline{l_{\nu L}^e}$	$\overline{l_L^e}$	$\overline{(l^c)^e}$	$\overline{l_{\nu L}^\mu}$	$\overline{l_L^\mu}$	$\overline{(l^c)^\mu}$	$\overline{l_{\nu L}^\tau}$	$\overline{l_L^\tau}$	$\overline{(l^c)^\tau}$
leptons	ν_{eR}	e_L^+	e_R	$\nu_{\mu R}$	μ_L^+	μ_L	$\nu_{\tau R}$	τ_L^+	τ_R

(4)The representations of the five A_0. Besides $T_0 = E$ is unit element, the other four algebras the generator is only one,it may correspond to the overall quantum number, such as the total spin S, baryon number, etc.Because of not space interval symmetry there is not parity of particles.

To sum up, we have the following analysis. (a) The basic irreducible representation of group algebra $A(T_d)$ exist and are the 18 lepton states.Their status are equivalent to the status of Gell-mann performed in the theory of quark with color (color here is $e\mu\tau$).(b) Because the O_h group algebra contains space inversion, the left-hand particle and right-hand particle exist equal.But the space inversion is not included in the group algebra $A(T_d)$,it leads to the asymmetry of particles and anti-particles.This is likely reason that there is the single color $(e\mu\tau)$ state in the group algebra $A(T_d)$ but not the single color (RBG) in the group algebra $A(O_h)$.

2. The multiplet of group algebra $A(T_d)$

Because of group algebra $A(T_d)$ and group algebra $A(O_h)$ isomorphism, therefore there are multiplet of $A(T_d)$.

(1)The multiplet

The multiplet of three flavors $(l\nu, l, l^c)_L$ are isomorphic to multiplet of three flavors (uds). These multplet in tensor $T_{ab...c}^{ij...k}$ are as Table X.

TABLE X. Flavor irreducible representation of group algebra

Irreducible tensor	1	T^i	T_a	T_a^i	T^{ij}	T_{ab}	T^{ijk}	T_{abc}
Class	(0, 0)	(1, 0)	(0, 1)	(1, 1)	(2, 0)	(0, 2)	(3, 0)	(0, 3)
Representation	1	3	$\underline{3}$	8	6	$\underline{6}$	10	$\underline{10}$

We first discuss the representation for dimension 8. The pseudo octet of group algebra $A(T_d)$ (spin $J = 0$) is below,

$$M(J=0) = \begin{pmatrix} l_{\nu L}\overline{l_{\nu L}} & l_{\nu L}\overline{l_L} & l_{\nu L}\overline{l_L^c} \\ l_L\overline{l_{\nu L}} & l_L\overline{l_L} & l_L\overline{l_L^c} \\ l_L^c\overline{l_{\nu L}} & l_L^c\overline{l_L} & l_L^c\overline{l_L^c} \end{pmatrix}. \tag{53}$$

And vector octet is

$$M(J=1) = \begin{pmatrix} l_{\nu L}\overline{l_{\nu L}} & l_{\nu L}\overline{l_L} & l_{\nu L}\overline{l_L^c} \\ l_L\overline{l_{\nu L}} & l_L\overline{l_L} & l_L\overline{l_L^c} \\ l_L^c\overline{l_{\nu L}} & l_L^c\overline{l_L} & l_L^c\overline{l_L^c} \end{pmatrix}, \tag{54}$$

and two single states are

$$\eta(J=0,1) = \frac{1}{\sqrt{3}}(l_{\nu L}\overline{l_{\nu L}} + l_L\overline{l_L} + l_L^c\overline{l_L^c}). \tag{55}$$

These representations $(l_{\nu L}^i, l_L^i, l_L^{ci})$,$(i = 1, 2, 3)$ belong to three basis representation.Can be done in the same way for higher dimension.But the most possible is in low dimensional state.Which particles is the state corresponding with the experiment?After considering the color $(e\mu\tau)$ answer again.

(2)The single state and multiplet of color $(e\mu\tau)$.Only with the color and flavor all (including spin) may be called particles.Above flavor multiplet has no color, are also not particle state.Because there is a single color state of group algebra $A(T_d)$, the flavor multiple state can be a colorless state (three color $e\mu\tau$ completely symmetrical) and also a color state.And there are color multiplet as well as flavor multiplet similar to the Table X.So particles of group algebra $A(T_d)$ are more than particles of group algebra $A(O_h)$ (three color completely symmetrical). However, considering the color is contact with strong interaction, color multiplet energy will be high, generally can not meet the existing experiment area.Therefore, we only consider the primary state, namely, $(l_{\nu L}, l_L, l_L^c)$ with single color $(e\mu\tau)$.

(3) Compared with the experiment particles

(3-1)The multiplet of single color e.(1)First,let to see the spin 0 pseudo-octet of Formula(53), their quantum number of each state(Y^d, I_3^d, Q) are

$$M(J=0) = \begin{pmatrix} (0,0,0) & (0,1,1) & (1,\frac{1}{2},-1) \\ (0,-1,-1) & (0,0,0) & (1,-\frac{1}{2},-2) \\ (-1,-\frac{1}{2},-1) & (-1,-\frac{1}{2},2) & (0,0,0) \end{pmatrix}. \tag{56}$$

We will fill in the experiment found that the so-called charm of mesons in Formula (56), and have

$$M(J=0)=\begin{pmatrix} \frac{D^0}{\sqrt{2}}-\frac{D^{0*}}{\sqrt{6}} & D^+ & F^- \\ D^- & -\frac{D^0}{\sqrt{2}}-\frac{D^{0*}}{\sqrt{6}} & F^{--} \\ F^+ & F^{++} & 2\frac{D^0}{\sqrt{6}} \end{pmatrix}. \tag{57}$$

Because there is the strong interaction of the T_d particles (in next section of this paper),we can get mass formula of spin zero the pseudo-octet in the same method with quark theory[41, 42] below.

$$3m_{D_{0*}}^2-4m_F^2+m_D^2=0. \tag{58}$$

In Formula (57),only $F^{\pm 2}, D^{0*}$ have not found.Because $m(D)=1869.3\pm 0.5MeV, m(F^{\pm}(D_s^{\pm}))=1968\pm 0.7MeV$,we can predict $m(F^{\pm 2})\approx 1970MeV, m(D^{0*})\approx 2000MeV$ from Formula(58).From Reference[16]the $D^{0*}(2010)$ may be mix with D^{0*}.

(2) Then see the vector octet in Formula (54).Their quantum number (Y^d, I_3^d, Q) is also Formula (56) with $spin1$.Let $J/\Psi, D^*(2010)^{\pm}, D^*(2010)^0, D_s^*(2010)^{\pm}$ to fill in it,we have

$$M(J=0)=\begin{pmatrix} \frac{D^{*0}(2010)}{\sqrt{2}}-\frac{J/\Psi}{\sqrt{6}} & D^{*+}(2010) & D_s^{*-} \\ D^{*-}(2010) & -\frac{D^{*0}(2010)}{\sqrt{2}}-\frac{J/\Psi}{\sqrt{6}} & F^{--} \\ F^+ & F^{++} & 2\frac{J/\Psi}{\sqrt{6}} \end{pmatrix}. \tag{59}$$

In additional to $F^{\pm 2}(J=1)$,other particles have found.Same consideration of quarks (uds) theory,we get $m(F^{\pm 2})\sim m(D_s^*(2110))\sim 2110MeV$.

(3) Particle $\eta_c(2980)$ can be filled in single state (J=0) of Formula(55);$\Psi(3685)$ in single state (J=1) of Formula(55).

(3-2) The multiplet with single color μ.

Because of $m_\mu=207m_e$,the multiplet with single color μ have more higher energy.There is more less data.Until 1992[16]possible particle are b quark meson $B^{\pm,0}(5278), B^*(5324), \Upsilon(9460)$,baryon $\Lambda_b^0(5641)$.Where $B^{\pm,0}(5278), B^*(5324)$ can fill in M^μ in Formula(53),the pseudo-meson octet with single color μ,

$$M(J=0)=\begin{pmatrix} \frac{B^0(5278)}{\sqrt{2}}-\frac{B^*(5324)}{\sqrt{6}} & B^+(5278) & F_b^- \\ B^-(5278) & -\frac{B^0(5278)}{\sqrt{2}}-\frac{B^*(5324)}{\sqrt{6}} & F_b^{--} \\ F_b^+ & F_b^{++} & 2\frac{B^*(5324)}{\sqrt{6}} \\ , & & \end{pmatrix}. \tag{60}$$

Where $F_b^{\pm 2}, F_b^{\pm 1}$ have not reported. Same considering in quark (uds) theory([41, 42]),their mass formula is following,

$$3m_{B^*(5324)}^2-4m_{F_b}^2+m_{B(5278)}^2=0. \tag{61}$$

In error 8 percent we predict $m(F_b)\approx(5300\pm 500)Mev$.

(3-3) The multiplet with single color τ.

Similar to color μ,because of $m_\tau\approx 17m_\mu$, it is hard to find the multiplet with single color τ in experiment.Until 1992[16],there is not report of this kind particles.

(4)Brief summary

By comparison with the experiment, the so-called (cbt) quarks particles can be replaced by the particle of the group algebra $A(T_d)$, then with colored quark is 18 lepton (see Table VIII and Table IX).Therefore, we do not need to use (cbt) quarks.Used in the above discussion of some of the symbols associated with the (cbt), just due to historical reasons, is advantageous for the textual research.

C. Conclusion

To this, we completely construct the all particles of existing experiments in the representation of group algebra $A(O_h)$ and group algebra $A(T_d)$,which is good coincidence to the experimental data, also predicted some the particles not yet found in the experiments.In this way, all the particles and interactions connected closely with spacetime structure in Formula (16), confirmed the thoughts that the existence of particles depends entirely on the structure of spacetime.

V. INTERACTIONS

In the matter spacetime, the particle states satisfy the symmetry Formula (16) in Section II, namely

$$\Psi'(\vec{r},t)=\exp[-i\theta^\alpha(\vec{r},t)T_\alpha]\Psi(\vec{r},t). \tag{62}$$

Here, T_α are the group elements (i.e.,generators),they are the point group symmetry elements of the hole or gap cell. According to the study in Section IV,(uds) quark particles,their symmetry is O_h, particles for hole O_h symmetry, antiparticle (uds) for the symmetry of the octahedral gap O_h.For $(l_{\nu L}, l_L l_L^c)$ leptons, their symmetry is T_d, the T_d symmetry of particle for hole state and antiparticle for tetrahedral gap symmetry of T_d. $\theta^\alpha(\vec{r},t)$ are the angle corresponding to each group elements T_α, it is always variant with spacetime coordinates.According to the local gauge field theory, Formula (62) corresponds to the local gauge transformation, T_α are the generators of the gauge group.The algebra of these gauge field generators T_α is the group algebra of O_h or T_d.Due to the local gauge field theory has developed quite perfect, our task is to make sure these generators to meet what kind of algebra, and bring a gauge field for every generator.I found that the group algebra of T_α is Lie algebra, can be decomposed into a number of irreducible Lie algebra. Corresponding to this a few irreducible Lie algebra, can introduce several kinds of gauge field, such as group algebra $A(T_d)$ (their representations are leptons).It can be decomposed into the direct sum of five Lie algebra A_0 , one Lie algebra A_1 ,two different Lie algebraA_2 (see Appendix). Based on the results of the structure of the particles in Section IV,the gauge field corresponding to the Lie sub-algebra is the interaction between particles(representations of corresponding Lie sub-algebra).The representation of A_2 is the flavor $(l_{\nu L}, l_L l_L^c)$,the eight gauge field of A_2 is interaction between them. The representation of A_2' is the color $(e\mu\tau)$,the eight gauge field of A_2' is interaction between them. The representation of A_1 is the spin,the three gauge field of A_1 is interaction between spin state,which is also kind of interaction.

The group algebra $A(O_h)$ is an expansion of group algebra $A(O)$, can be decomposed into direct sum of left-hand group algebra $A(O^+)$ and right-hand group algebra $A(O^-)$ (see Formula (20) and (36)), explicitly write as follows:

$$A(O) = \underbrace{\sum_{i=1}^{5} \oplus A_0^i \oplus A_1 \oplus A_2 \oplus A_2' \oplus}_{A(O^+)} \underbrace{\sum_{i=1}^{5} \oplus A_0^i \oplus A_1 \oplus A_2 \oplus A_2'}_{A(O^-)}. \tag{63}$$

where

$$\begin{cases} T_L \equiv T^+ = \frac{(1+\sigma)}{2} T_i \in A(O_h^+), T_i \in A(O), \\ T_R \equiv T^- = \frac{(1-\sigma)}{2} T_i \in A(O_h^-). \end{cases} \tag{64}$$

This makes the representation $\Psi(\vec{r},t)$ of $A(O_h)$ is decomposed into $\Psi_L(\vec{r},t)$ and $\Psi_R(\vec{r},t)$.They are respectively in the Dirac representation,

$$\begin{cases} \Psi_L(\vec{r},t) = T_L \Psi(\vec{r},t) = \frac{(1+\gamma_5)}{2} \Psi(\vec{r},t), \\ \Psi_R(\vec{r},t) = T_R \Psi(\vec{r},t) = \frac{(1-\gamma_5)}{2} \Psi(\vec{r},t). \end{cases} \tag{65}$$

Where $\Psi(\vec{r},t)$ is the representation of group algebra $A(O)$, the γ_5 is a Dirac matrix. So Ψ_L and Ψ_R correspond to the left-hand and right-hand representation of group algebra $A(O_h)$.

By Formula (62), $T_\alpha \in O_h$ can be decomposed into two parts T_α and σT_α and their corresponding angle are $\gamma^\alpha(\vec{r},t)$ and $\gamma_\sigma^\alpha(\vec{r},t)$ respectively.Without considering deformation of cell,it is easy to find from below Fig.9,

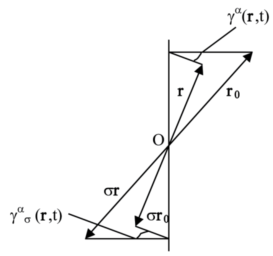

Fig. 9 $\gamma^\alpha(\mathbf{r},t)$ and $\gamma^\alpha_\sigma(\mathbf{r},t)$

$$\begin{cases} \gamma^\alpha(\vec{r},t) = \gamma_\sigma^\alpha(\vec{r},t), \\ \theta^\alpha(\vec{r},t) = \theta_\sigma^\alpha(\vec{r},t). \end{cases} \tag{66}$$

Using Formula(66) the Formula (62) becomes

$$\Psi'(\vec{r},t) = \exp[-i\theta^\alpha(\vec{r},t)T_\alpha - i\theta_\sigma^\alpha(\vec{r},t)\sigma T_\alpha](\Psi_L + \Psi_R) \tag{67}$$

$$= \exp[-i2\theta^\alpha(\vec{r},t)T_\alpha]\Psi_L(\vec{r},t) + \Psi_R(\vec{r},t). \tag{68}$$

This suggests that: (1) Interaction occurs only between left-handed state, and right-hand part does not interact with left-hand part, and no interaction between the right-hand part,this is equivalent to say an interaction is pure $V - A$ type, no right current.It is not coincident the results of existing experiment neutral current (such as electromagnetic current)[44, 45], and the QCD is not pure $V - A$ theory.(2) Both the gauge field theory of group algebra $A(T_d)$ and group algebra $A(O_h)$,the particle mass and gauge particle mass are all zero.It also does not accord with the existing experiment.The gauge particle mass of flavor interaction is $m(W^\pm, Z^0) \sim 10^2 GeV$.

The reason why are there above two results, is that we didn't consider the deformation of cell in the previous discussion.There are the different degree of deformation of every cell in the matter spacetime.I will consider the cell deformation in matter spacetime (i.e.,particle field),which is called generalized Higgs mechanism.Now, let's look at how to describe the deformation.

First, because the cell deformation, Formula (62) will no longer be established for all .This is likely to solve the above the first contradiction.

Second, for each cell $(\vec{r},t)$, its geometry shape can use a function $F(\vec{r},t)$ to describe, and $F(\vec{r},t)$ always can be expanded in the following form

$$F(\vec{r},t) = l(\vec{r},t) + C_\mu(\vec{r},t)\frac{\partial F(\vec{r},t)}{\partial x^\mu} + \frac{1}{2!}C_{\mu\nu}(\vec{r},t)\frac{\partial^2 F(\vec{r},t)}{\partial x^\mu \partial x^\nu} + ... \tag{69}$$

This suggests that the cell deformation of any position can be described by the scalar field $l(\vec{r},t)$, vector field $C_\mu(\vec{r},t)$ and other tensor field.Only for a scalar field $l(\vec{r},t)$, if you place it as the scale of the cell $(\vec{r},t)$ (such as the diameter), their vacuum value is not zero, that is characteristic of the Higgs field[36, 37].It has the potential to solve the above second contradiction.So these fields called generalized Higgs fields.

A. Interactions of $A(O_h)$ particles :flavor,color,spin interactions

This family of particles have the symmetry $S(O_h)$ (defined in Formula (10)) with the group algebra $A(O_h)$.The $A(O_h)$ can be decomposed into the direct sum of some Lie algebra.

1. *Flavor dynamics*

We discuss first at the interaction between flavor (uds) representations.In the original concept of *internal* symmetry, it is equivalent to the internal symmetry of the $SU_F(3)$ flavor.According to Formula (62),there are only interaction between $(uds)_L$, and not between $(uds)_R$, and not between $(uds)_L$ and $(uds)_R$.

There are eight generators of A_2.Their relationship between the $T_\alpha \in O$ and the standard generators of A_2 is Formula (34).In order to contrast to the GWS (standard) model, we choose eight Gell-mann matrix as the generators of Lie algebra A_2, T_i,i=1,2,...,8.

$$T_1=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix},\quad T_2=\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix},$$
$$T_3=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix},\quad T_4=\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix},$$
$$T_5=\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}\quad T_6=\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix},$$
$$T_7=\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix},\quad T_8=\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}.$$

Therefore Formula (62) becomes

$$\Psi'(\vec{r},t)=\exp[-i\omega^i(\vec{r},t)T_i]\Psi_L(\vec{r},t)+\Psi_R(\vec{r},t). \tag{70}$$

Where $T_i(i=1,2,...,8)$ are the Gell-mann matrixes,Ψ_L and Ψ_R are the representations of A_2,$\omega^i(\vec{r},t)$ is a function of $\theta(\vec{r},t)$,$\omega^i(\vec{r},t)$ is also the function of spacetime $(\vec{r},t)$,therefore Formula(70) is a local gauge transformation.

According to the local gauge invariance theory, there are the gauge field $A_\mu(\vec{r},t)$ for every generator T_i,namely

$$T_i, A^i_\mu(\vec{r},t), i=1,2,...,8. \tag{71}$$

There are eight gauge fields, which are the function of spacetime coordinate.

Let

$$\frac{1}{\sqrt{2}}A_\mu(\vec{r},t)\equiv\frac{1}{2}\sum_{i=1}^{8}T_iA^i_\mu(\vec{r},t), \tag{72}$$

we have

$$A_\mu(\vec{r},t)=\begin{pmatrix}\frac{A^3_\mu}{\sqrt{2}}+\frac{A^8_\mu}{\sqrt{6}} & \frac{A^1_\mu-iA^2_\mu}{\sqrt{2}} & \frac{A^4_\mu-iA^5_\mu}{\sqrt{2}}\\ \frac{A^1_\mu+iA^2_\mu}{\sqrt{2}} & -\frac{A^3_\mu}{\sqrt{2}}+\frac{A^8_\mu}{\sqrt{6}} & \frac{A^6_\mu-iA^7_\mu}{\sqrt{2}}\\ \frac{A^4_\mu-iA^5_\mu}{\sqrt{2}} & \frac{A^6_\mu+iA^7_\mu}{\sqrt{2}} & -2\frac{A^8_\mu}{\sqrt{6}}\end{pmatrix}. \tag{73}$$

It is a 3×3 matrix $A^b_{\mu a}(\vec{r},t)$,(a,b=1,2,3),which gauge field satisfies following formula,

$$\sum_{a=1}^{3}A^a_{\mu a}(\vec{r},t)=0. \tag{74}$$

Do following transformation,

$$\begin{aligned}W_{1\mu}&=\tfrac{A^1_\mu+iA^2_\mu}{\sqrt{2}}, & \overline{W_{1\mu}}&=\tfrac{A^1_\mu-iA^2_\mu}{\sqrt{2}},\\ W_{2\mu}&=\tfrac{A^4_\mu+iA^5_\mu}{\sqrt{2}}, & \overline{W_{2\mu}}&=\tfrac{A^4_\mu-iA^5_\mu}{\sqrt{2}},\\ X_\mu&=\tfrac{A^6_\mu+iA^7_\mu}{\sqrt{2}}, & \overline{X_\mu}&=\tfrac{A^6_\mu-iA^7_\mu}{\sqrt{2}},\\ X'_\mu&=-\tfrac{A^3_\mu+\sqrt{3}A^7_\mu}{2}, & B_\mu&=\tfrac{\sqrt{15}}{6}(-A^3_\mu-\tfrac{A^8_\mu}{\sqrt{3}}).\end{aligned}$$

The gauge field matrix becomes

$$A_\mu(\vec{r},t)=\begin{pmatrix}\frac{-6}{\sqrt{30}}B_\mu & \overline{W}_{1\mu} & \overline{W}_{2\mu}\\ W_{1\mu} & \frac{1}{2}X'_\mu+\frac{3}{\sqrt{30}}B_\mu & \overline{X}_\mu\\ W_{2\mu} & X_\mu & -\frac{1}{2}X'_\mu+\frac{3}{\sqrt{30}}B_\mu\end{pmatrix}. \tag{75}$$

According to the general gauge field,for the Fermi field,

$$\Psi(\vec{r},t)=\begin{pmatrix}u\\d\\s\end{pmatrix},$$

the gauge invariance lagrangian from gauge transformation Formula (62) or (70) are

$$\pounds_f=i\Psi\overline{D}\Psi=i\Psi_L\overline{D}_1\Psi_L+i\Psi_R\overline{D}_2\Psi_R, \tag{76}$$

where $\overline{D},\overline{D}_1,\overline{D}_2$ are covariant derivations,they are

$$\overline{D}=\gamma_\mu D_\mu,\overline{D}_1=\gamma_\mu D_{1\mu},\overline{D}_2=\gamma_\mu D_{2\mu}, \tag{77}$$

$$D_{1\mu}=\partial_\mu-ig_FT_iA^i_\mu(\vec{r},t),D_{2\mu}=\partial_\mu, \tag{78}$$

g_F is a coupling constant.The Formula (76) becomes

$$\begin{aligned}\pounds_f=&i\begin{pmatrix}\overline{u}&\overline{d}&\overline{s}\end{pmatrix}\gamma_\mu\partial_\mu\begin{pmatrix}u\\d\\s\end{pmatrix}\\ &+\frac{g_F}{\sqrt{2}}\begin{pmatrix}\overline{u}&\overline{d}&\overline{s}\end{pmatrix}_L\gamma_\mu A_\mu(\vec{r},t)\begin{pmatrix}u\\d\\s\end{pmatrix}_L.\end{aligned} \tag{79}$$

Therefore the interaction term between the gauge fields and particles is

$$\pounds_{fg}=\frac{g_F}{\sqrt{2}}\begin{pmatrix}\overline{u}&\overline{d}&\overline{s}\end{pmatrix}_L\gamma_\mu A_\mu(\vec{r},t)\begin{pmatrix}u\\d\\s\end{pmatrix}_L. \tag{80}$$

From Formula(79) and (80) we can get that: (1) The quarks (uds) and gauge field have no mass term.(2) The interaction between the gauge fields and particles is pure V-A type.But the charge current is pure V-A type, and type $(\bar{u}u)$ neutral current is not pure V-A type in experiment[44, 45].Therefore,for the charge current Formula (80) coincides with experiment,for the neutral current this theory does not coincide with the experiment.(3)This theory predicts that exist $(\bar{d}s)$ neutral current. The experiment results show that the neutral current is smaller than $(\bar{u}u)$ neutral current in eight orders of magnitude[40].In the GWS (standard) model through GIM mechanism[28] the $(\bar{d}s)$ and $(\bar{s}d)$ neutral current will be take out,such current doesn't exist.Above these three questions can be solved through the cell deformation (generalized Higgs mechanism).

2. Generalization Higgs mechanism

After considering the cell deformation the Formula (66) will no longer be set up.If the deformation is small in the weak interaction only in T_8,

$$\omega^i(\vec{r},t) = \omega^i(\vec{r},t), i = 1,2,...,7; \omega^8(\vec{r},t) \neq \omega^8(\vec{r},t). \quad (81)$$

Then Formula (62) will be

$$\begin{aligned}\Psi'(\vec{r},t) =& \exp[-i\omega^i(\vec{r},t)T_i - i\eta Y]\Psi_L(\vec{r},t) \\ &+ \exp[-i\eta Y]\Psi_R(\vec{r},t),\end{aligned} \quad (82)$$

where

$$Y = \frac{1}{\sqrt{3}}T_8, \eta(\vec{r},t) = \eta(\theta^8(\vec{r},t) - \theta^8_\sigma(\vec{r},t)). \quad (83)$$

According to the gauge field theory, the Formula (82) shows that the interaction between particles is the local gauge invariant theory. In addition to the interaction $A_\mu(\vec{r},t)$ in the Formula (81), we need a gauge filed $\Phi_\mu(\vec{r},t)$ for the Y, namely

$$Y, \Phi_\mu(\vec{r},t) \quad (84)$$

So the covariant derivative of $\Psi_L(\vec{r},t)$ is follows,

$$D_{1\mu} = \partial_\mu - ig_F A^i_\mu(\vec{r},t)T_i - ig'_F\Phi_\mu(\vec{r},t)Y. \quad (85)$$

The covariant derivative of $\Psi_R(\vec{r},t)$ is follows,

$$D_{2\mu} = \partial_\mu - ig'_F\Phi_\mu(\vec{r},t)Y. \quad (86)$$

So there are two gauge fields: $A^i_\mu(\vec{r},t)$ and $\Phi_\mu(\vec{r},t)$,the total gauge field intensity is the sum of the following two:

$$\begin{cases} F_{\mu\nu} = (\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)T_i + g_F f^k_{ij}A^i_\mu A^j_\nu T_k, \\ B_{\mu\nu} = \partial_\mu\Phi_\nu - \partial_\nu\Phi_\mu. \end{cases} \quad (87)$$

The Lagrangian of the total gauge field is follows,

$$\pounds_g = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}. \quad (88)$$

According to the gauge field theory, after considering the cell deformation, the Lagrangian in Formula (76) becomes

$$\pounds_f = i\bar{\Psi}_L\gamma_\mu D_{1\mu}\Psi_L + i\bar{\Psi}_R\gamma_\mu D_{2\mu}\Psi_R. \quad (89)$$

Here $D_{1\mu}, D_{2\mu}$ are Formula (85) and (86).Note Ψ_L is a three-dimensional representation, $I_3 \neq 0$ but Ψ_R are three dimensional representations, $I_3 = 0$.By the Gell-mann-Nishijima Formula, we have:

$$\begin{cases} Y_L = 2(Q - I_3), \\ Y_R = 2Q. \end{cases} \quad (90)$$

So Formula (89) can be explicitly written,

$$\pounds_f = i\begin{pmatrix} \bar{u} & \bar{d} & \bar{s} \end{pmatrix}\gamma_\mu\partial_\mu\begin{pmatrix} u \\ d \\ s \end{pmatrix} + \pounds^{(eff)}_{fg}. \quad (91)$$

Where the interaction term $\pounds^{(eff)}_{fg}$ is follows,

$$\pounds^{(eff)}_{fg} = \frac{g_F}{\sqrt{2}}\begin{pmatrix} \bar{u} & \bar{d} & \bar{s} \end{pmatrix}\gamma_\mu A^{eff}_\mu(\vec{r},t)\begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad (92)$$

where

$$A^{(eff)}_\mu(\vec{r},t) = \begin{pmatrix} R_{1\mu} & \frac{1+\gamma_5}{2}\overline{W}_{1\mu} & \frac{1+\gamma_5}{2}\overline{W}_{2\mu} \\ \frac{1+\gamma_5}{2}W_{1\mu} & R_{2\mu} & \frac{1+\gamma_5}{2}\overline{X}_\mu \\ \frac{1+\gamma_5}{2}W_{2\mu} & \frac{1+\gamma_5}{2}X_\mu & R_{3\mu} \end{pmatrix}, \quad (93)$$

$$\begin{aligned}R_{1\mu} &= \frac{1+\gamma_5}{2}(\frac{1}{\sqrt{2}}A^3_\mu + \frac{1}{\sqrt{6}}A^8_\mu + \frac{g'_F}{3g_F}\Phi_\mu) + \frac{1-\gamma_5}{2}\frac{g'_F}{3g_F}\Phi_\mu, \\ R_{2\mu} &= \frac{1+\gamma_5}{2}(-\frac{1}{\sqrt{2}}A^3_\mu + \frac{1}{\sqrt{6}}A^8_\mu + \frac{g'_F}{3g_F}\Phi_\mu) - \frac{1-\gamma_5}{2}\frac{g'_F}{3g_F}\Phi_\mu, \\ R_{3\mu} &= \frac{1+\gamma_5}{2}(-\frac{2}{\sqrt{6}}A^8_\mu - \frac{2g'_F}{3g_F}\Phi_\mu) - \frac{1-\gamma_5}{2}\frac{2g'_F}{3g_F}\Phi_\mu \\ &. \end{aligned} \quad (94)$$

The effective interaction in Formula (93) shows that the charge current is pure V-A type, and neutral current is the V-A type, but is no longer the pure V- A type. The result coincides with the experiment. Only consider type neutral current j^{NC} first.

$$\begin{aligned}j^{NC}_\mu =& \frac{1+\gamma_5}{2}(\sqrt{2}j^I A^3_\mu + \frac{3}{\sqrt{6}}j^Y A^8_\mu + \frac{g'_F}{g_F}j^Y\Phi_\mu) \\ &+ \frac{1-\gamma_5}{2}\frac{g'_F}{g_F}j^Y\Phi_\mu,\end{aligned} \quad (95)$$

where

$$j^I = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad j^Y = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}, \quad Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}. \quad (96)$$

In Formula(95) there is a kind of neutral current,it is the charge current eQA_μ,without loss of generality,do transformation as below[40],

$$\begin{pmatrix} \Phi_\mu \\ A^8_\mu \\ A^3_\mu \end{pmatrix} = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_1 & c_1c_2c_3 + s_2s_3 & c_1c_2s_3 - s_2c_3 \\ s_1s_2 & -c_1s_2c_3 - c_2s_3 & -c_1s_2c_3 + c_2c_3 \end{pmatrix}\begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}, \quad (97)$$

here $c_i = cos\theta_i, s_i = sin\theta_i, i = 1,2,3$.We have neutral current j^{NC},

$$j^{NC(q\bar{q})}_\mu = j_A A_\mu + j_Z Z_\mu + j_{Z'}Z'_\mu. \quad (98)$$

Where

$$\begin{aligned}j_A =& \frac{1+\gamma_5}{2}[\sqrt{2}j^I(-\sin\theta_1\cos\theta_2) + j^Y(\frac{3}{\sqrt{6}}\sin\theta_1\sin\theta_2 \\ &+ \frac{g'_F}{g_F}\cos\theta_1)] + \frac{1-\gamma_5}{2}\frac{g'_F}{g_F}Q\cos\theta_1,\end{aligned} \quad (99)$$

$$j_Z = \frac{1+\gamma_5}{2}\{\sqrt{2}j^I(\cos\theta_1 cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3) + j^Y[\frac{3}{\sqrt{6}}(-\cos\theta_1\sin\theta_2\cos\theta_3 - \cos\theta_2\sin\theta_3) + \frac{g'_F}{g_F}\sin\theta_1\cos\theta_3]\} + \frac{1-\gamma_5}{2}\frac{g'_F}{g_F}Q\sin\theta_1\sin\theta_3, \tag{100}$$

$$j_{Z'} = \frac{1+\gamma_5}{2}\{\sqrt{2}j^I(\cos\theta_1 cos\theta_2\sin\theta_3 + \sin\theta_2\cos\theta_3) + j^Y[\frac{3}{\sqrt{6}}(-\cos\theta_1\sin\theta_2\sin\theta_3 + \cos\theta_2\cos\theta_3) + \frac{g'_F}{g_F}\sin\theta_1\sin\theta_3]\} + \frac{1-\gamma_5}{2}\frac{g'_F}{g_F}Q\sin\theta_1\sin\theta_3. \tag{101}$$

If $A(\vec{r},t)$ is electromagnetic field,we requires

$$g_F j_A = eQ. \tag{102}$$

Consider Formula (99) with Formula (102),we have

$$\sin\theta_1 = \frac{\sqrt{2}g'_F}{\sqrt{3g_F^2+2g_F'^2}}, \sin\theta_2 = -\frac{1}{2}, \tag{103}$$

$$\cos\theta_2 = -\frac{\sqrt{3}}{2}, \frac{3g_F g'_F}{\sqrt{\sqrt{3}g_F^2+2g_F'^2}} = e. \tag{104}$$

Using Formula (96),(103) and (104) in Formula (100) and (101),we get the matrix of $j_Z, j_{Z'}$ follows.

$$j_Z = \begin{pmatrix} a_1+b_1\gamma_5 & 0 & 0 \\ 0 & a_2+b_2\gamma_5 & 0 \\ 0 & 0 & a_3+b_3\gamma_5 \end{pmatrix}, \tag{105}$$

$$j_{Z'} = \begin{pmatrix} c_1+d_1\gamma_5 & 0 & 0 \\ 0 & c_2+d_2\gamma_5 & 0 \\ 0 & 0 & c_3+d_3\gamma_5 \end{pmatrix}, \tag{106}$$

where

$$\begin{cases} a_1 = \frac{e}{\sqrt{6}}g'_F(\frac{g_F'^2}{g_F^2}-\frac{1}{2})\cos\theta_3 + \frac{\sqrt{2}}{4}\sin\theta_3, \\ b_1 = \frac{e}{3\sqrt{6}}g'_F(\frac{g_F'^2}{g_F^2}-\frac{3}{2})\cos\theta_3 + \frac{\sqrt{2}}{4}\sin\theta_3, \\ a_2 = \frac{e}{\sqrt{6}}g'_F\cos\theta_3, \\ b_2 = \frac{e}{3\sqrt{6}}g'_F(\frac{g_F'^2}{g_F^2}+\frac{3}{2})\cos\theta_3, \\ a_3 = \frac{-e}{\sqrt{6}}g'_F(\frac{g_F'^2}{g_F^2}+\frac{1}{2})\cos\theta_3 - \frac{\sqrt{2}}{4}\sin\theta_3, \\ b_3 = \frac{-e}{3\sqrt{6}}g'_F(\frac{g_F'^2}{g_F^2}+\frac{3}{2})\cos\theta_3 - \frac{\sqrt{2}}{4}\sin\theta_3, \\ c_1 = \frac{e}{\sqrt{6}}g'_F(\frac{g_F'^2}{g_F^2}-\frac{1}{2})\sin\theta_3 - \frac{\sqrt{2}}{4}\cos\theta_3, \\ d_1 = \frac{-e}{3\sqrt{6}}g'_F(\frac{g_F'^2}{g_F^2}+\frac{3}{2})\sin\theta_3 - \frac{\sqrt{2}}{4}\cos\theta_3, \\ c_2 = \frac{e}{\sqrt{6}}g'_F\sin\theta_3, \\ d_2 = \frac{2e}{3\sqrt{6}}g'_F(\frac{g_F'^2}{g_F^2}+\frac{3}{2})\sin\theta_3, \\ c_3 = \frac{-e}{\sqrt{6}}g'_F(\frac{g_F'^2}{g_F^2}+\frac{1}{2})\sin\theta_3 + \frac{\sqrt{2}}{4}\cos\theta_3, \\ d_3 = \frac{-e}{3\sqrt{6}}g'_F(\frac{g_F'^2}{g_F^2}+\frac{3}{2})\sin\theta_3 + \frac{\sqrt{2}}{4}\cos\theta_3. \end{cases} \tag{107}$$

Above all g_F, g'_F, θ_3 can be determined by experiment.Because of the experimental results that current Z_μ is much larger than current Z'_μ we have

$$\begin{cases} \frac{a_1}{b_1} = 1 - \frac{8}{3}\sin^2\theta_w, \\ \frac{a_2}{b_2} = 1 - \frac{4}{3}\sin^2\theta_w. \end{cases} \tag{108}$$

Where θ_W is so called Weinberg Angle. In experiment,

$$\sin^2\theta_W = 0.224 \pm 0.02, e^2 = \frac{4\pi}{137}. \tag{109}$$

We got

$$\begin{cases} g_F = 0.3616, \\ g'_F = 0.4525, \\ \tan\theta_3 = -0.412, \\ \sin\theta_1 = 0.546. \end{cases} \tag{110}$$

Above discusses, because of the cell deformation, only interactions between particles considered. After considering the cell deformation themselves, the generalized Higgs field with Formula (69) should be introduced. Under the first approximation, we only introduce a scalar field $l(\vec{r},t)$.As this field is in the matter spacetime, the scalar field $l(\vec{r},t)$ shall meet the gauge invariance of Formula (62), namely, $l(\vec{r},t)$ is a scalar field in the spacetime, and also the representation of A_2 (equivalent to internal coordinates).In general, the gauge invariant Lagrangian of a scalar field is

$$\pounds = D^\mu l^+(\vec{r},t)D_\mu l(\vec{r},t) + U(l), \tag{111}$$

where

$$D_\mu = \partial_\mu - i\frac{g_F}{\sqrt{2}}A_\mu^{eff}(\vec{r},t). \tag{112}$$

In order to make gauge field mass, we can choose an appropriate potential function $U(l)$.Considering the $l(\vec{r},t)$ in a vacuum,

$$l(\vec{r},t)_{vacuum} = l_0, \tag{113}$$

the $l(\vec{r},t)$ has the characteristic of the Higgs field. According to the Higgs mechanism[36, 37],we can get the mass of $W_{1,2}^\pm, Z, Z'$, but not X, X'.There are two possible reasons for this. One is the difficulty of the Higgs mechanism itself, another is a research on Higgs mechanism study is not good yet. The following work is to estimate the vacuum l_0 by the experiment mass of $W^\pm, Z$. The scalar field $l(\vec{r},t)$ is the multiplet of A_2. First we choose 3-dimension basic representation $l^\alpha(\vec{r},t), \alpha = 1,2,3.$ and it is a typical Higgs potential,

$$U(l) = -\frac{1}{2}\mu^2(l^+l) + \frac{1}{4}\lambda(l^+l). \tag{114}$$

When

$$(l^+l) = \frac{\mu^2}{\lambda} \equiv \langle l^+l\rangle_0. \tag{115}$$

$U(l)$ take extreme value. We take vacuum average $\langle l^\alpha \rangle_0$ of l^α to

$$\begin{cases} \langle l^\alpha \rangle_0 = \begin{pmatrix} \frac{v_0}{\sqrt{2}} \\ 0 \\ 0 \end{pmatrix}, \\ v_0^2 = \frac{2\mu^2}{\lambda}. \end{cases} \tag{116}$$

Do a translation as bellows,

$$l^\alpha = \langle l^\alpha \rangle_0 + l'^\alpha. \tag{117}$$

We get the mass term of gauge field below:

$$\frac{1}{4} g_F^2 v_0^2 (A_\mu^{eff})'_\alpha (A_\mu^{eff})'^\alpha, \tag{118}$$

$\alpha = 1$ is the mass term of Z, Z', $\alpha = 2, 3$ are the mass terms of $W^\pm$ respectively. We got

$$M_{W,Z} = \frac{1}{2} g_F v_0 = \frac{1}{\sqrt{2}} g_F l_0. \tag{119}$$

Known $M_{W,Z} = 10^2 GeV, g_F \sim 0.45$,then

$$v_0 \sim 3 \times 10^2 GeV. \tag{120}$$

So

$$l_0 \sim 10^2 GeV. \tag{121}$$

Such l_0 is equivalent to length

$$l_0 \sim \frac{hc}{10^2 GeV} \sim 10^{-18} m. \tag{122}$$

This results are consistent with the estimate by Lee T.D.[34].I have tried to improve the Higgs method to get mass of $X, \overline{X}$ to meet the requirements of less neutral current of the $(\overline{d}s)$ and $(d\overline{s})$,but did not succeed. But I believe, to get fine mass of X is possible.

3. Color dynamics

If not considering the difference of $SU_c^L(3)$ (left-hand color states) and $SU_c^R(3)$ (right-hand color states), the color-dynamics here $SU(3)$ gauge theory is the same as the usual chromodynamics[46].So often chromodynamics conclusions can be get. However, there is difference of left-hand part and right-hand in my theory, which can be illustrated through the analysis of the following. In flavor dynamics, due to the weak interaction, the Formula (66) also founded approximation after considered the deformation.So,there is asymmetry of left-hand and right-hand in flavor interaction. However, in the color interaction (chromodynamics), the cell is with larger deformation and $\theta^\alpha(\vec{r}, t), \theta^\alpha_\sigma(\vec{r}, t)$ become two independent variables.From Formula (62) we have

$$\begin{aligned} &\exp[-i\theta^\alpha(\vec{r}, t)T_\alpha - i\theta^\alpha_\sigma(\vec{r}, t)\sigma T_\alpha]\Psi(\vec{r}, t) \\ &= \exp[-i\theta^\alpha_1(\vec{r}, t)T_\alpha]\Psi_L(\vec{r}, t) + \exp[-i\theta^\alpha_2(\vec{r}, t)T_\alpha]\Psi_R(\vec{r}, t). \end{aligned} \tag{123}$$

Here

$$\begin{cases} \theta^\alpha_1(\vec{r}, t) = \theta^\alpha(\vec{r}, t) + \theta^\alpha_\sigma(\vec{r}, t), \\ \theta^\alpha_2(\vec{r}, t) = \theta^\alpha(\vec{r}, t) - \theta^\alpha_\sigma(\vec{r}, t). \end{cases} \tag{124}$$

In this way, the color interaction is strictly symmetrical between the left-hand and right-hand. The eight generators of A'_2 and the eight gluon fields are one-to-one, $A_\mu(\vec{r}, t), (i = 1, 2, ..., 8; \mu = 0, 1, 2, 3)$.Its gauge field density is bellows.

$$F^i_{\mu\nu} = (\partial_\mu A^i_\nu - \partial_\nu A^i_\mu) + g_c f^i_{jk} A^j_\mu A^k_\nu. \tag{125}$$

The g_c for coupling constant, f^i_{jk} for structure constant of A'_2.So my color interaction theory and the usual chromodynamics are consistent. The difference is that in my theory in the use of the method of discrete processing, just as low energy, interaction distance is greater than l_0, can approximate method with continuous processing. This is why in the low energy region usually chromodynamics reasonable. Further discussion does not repeat here.

4. Spin dynamics

In my theory, there is an interaction between spin, and the status of spin interaction is same as the status of flavor interaction and color interaction. So it is also a kind of basic interactions. Strictly speaking, the interaction of spin and spin are the interaction of the third spin components, which is the each spin state of particles.

However, in our previous theory, spin interaction is contained in the electromagnetic interaction; spin interaction is not a kind of independent interactions. The success of quantum electrodynamics[47] makes that it is sure.

If we dig into the successful reason of quantum electrodynamics,the foundation is Dirac equation. It is Dirac equation which makes the quantum electrodynamics correction for the interaction between spin and electromagnetic field, spin and spin, was very precise results. Such as theoretical results of electronic magnetic moment consistent surprisingly with experimental results. But the Dirac equation is the most basic? Answer is that the Dirac equation is not the most basic, but the inevitable result of the space time invariance of the Poincare[48].The four component spin representation of Poincare group are $|\mathbf{k}^\alpha\rangle, (\alpha = 1, 2, 3, 4)$, their dot product $\mathbf{P} \cdot \mathbf{P}$ of momentum operator $\mathbf{P}$ is a Casmir operator, it led to the equation

$$\mathbf{P} \cdot \mathbf{P}|k\rangle = -k^2|k\rangle. \tag{126}$$

However, after considering spatial inversion theory of four components, $\mathbf{P}$ is no longer a Poincare invariant operator, the invariant operator is $\vec{\gamma} \cdot \mathbf{P}$,which $\vec{\gamma}$ are four Dirac matrices.We can prove

$$(\gamma \cdot \mathbf{P})^2 = \mathbf{P} \cdot \mathbf{P}. \tag{127}$$

So, combining Formula (126), the eigenvalue of the $\gamma \cdot \mathbf{P}$ is $\pm ik$. Thus we get the one constraint condition of the four component expression below,

$$(\gamma \cdot \mathbf{P} + ik)|k\rangle = 0. \tag{128}$$

It is just the Dirac equation (take the eigenvalue of the $\gamma \cdot \mathbf{P}$ for $-ik$).So,the Dirac equation is the inevitable result of the Poincare symmetry

If we noticed that above discussion is done in the usual four dimensional spacetime, will produce a problem that spin is generally a property of spacetime, like $\hat{k}$,is that right? The answer is: no.We explore the introducing process of spin in the above theory[48], and know that spin $S = \frac{1}{2}$ corresponds to the double value of the rotation, i.e., the space rotation of angle 2π does not return to zero angle position in the spacetime, but angle 4π came back to zero angle position ($0 \leq \varphi \leq 4\pi$).This requirement is unacceptable without considering the internal structure of spacetime. In other words, acceptation of the double value (spin) means that the structure of spacetime is considered. So the Dirac equation is the result of the internal structure of space time. This structure just appeared in my theory.Therefore, the results of quantum electrodynamics prove the existence of the interaction between the spin. In the following we discuss that the spin interaction is essential.

According to gauge theory, considering the local gauge field of spin algebra A_1 and flavor algebra A_2, then

$$D_\mu = \partial_\mu - ig_F A^\alpha_\mu(\vec{r},t)T_\alpha - ig_s B^\gamma_\mu(\vec{r},t)\tau_\gamma. \tag{129}$$

Where g_F, g_s are two coupling constants, $T_\alpha \in A_2, \tau_\gamma \in A_1.B^\gamma_\mu(\vec{r},t)$ is gauge field corresponding to spin generator τ_γ. Define

$$\begin{aligned} A_\mu(\vec{r},t) &= A_\mu(\vec{r},t) + \frac{g_S}{g_F} B_\mu(\vec{r},t) \\ &= A^\alpha_\mu(\vec{r},t)T_\alpha + \frac{g_S}{g_F} B^\gamma_\mu(\vec{r},t)\tau_\gamma. \end{aligned} \tag{130}$$

Formula (129) becomes

$$D_\mu = \partial_\mu - ig_F A'^\alpha_\mu(\vec{r},t)T_\alpha. \tag{131}$$

Here the difference between $A'^\alpha_\mu(\vec{r},t)$ and $A^\alpha_\mu(\vec{r},t)$ is : $A^\alpha_\mu(\vec{r},t)$ is only gauge field with the particle flavor index, doesn't work for the spin index, and $A'^\alpha_\mu(\vec{r},t)$ is gauge field with the particle flavor index and spin index. That is to say, $A^\alpha_\mu(\vec{r},t)$ is pure interaction between flavors, and $A'^\alpha_\mu(\vec{r},t)$ is interaction between flavors and the interaction between spin. If a particle is with flavor and spin, the interaction between them is $A'^\alpha_\mu(\vec{r},t)$.This is the results in flavor dynamics (of course, is also the result of the in color dynamics).For example, in the interaction between flavor u,

$$\frac{g_F}{\sqrt{2}} \overline{u} \gamma^\mu Z_\mu u. \tag{132}$$

If we do not consider the spin orientation of u, we don't know spin component m_z of Z_μ, it doesn't matter spin of Z_μ.However, when we consider the direction of the spin, we have the following four terms,

$$\begin{array}{ll} \frac{g_F}{\sqrt{2}} \overline{u}_\uparrow \gamma^\mu Z_\mu u_\uparrow, & \frac{g_F}{\sqrt{2}} \overline{u}_\downarrow \gamma^\mu Z_\mu u_\downarrow, \\ \frac{g_F}{\sqrt{2}} \overline{u}_\uparrow \gamma^\mu Z_\mu u_\downarrow, & \frac{g_F}{\sqrt{2}} \overline{u}_\downarrow \gamma^\mu Z_\mu u_\uparrow. \end{array} \tag{133}$$

While in these four terms we use the same symbol Z_μ, but as we know, m_z of Z_μ in each terms are different to each other. In the first two terms, $m_z = 0$,in later two terms, $m_z = \pm 1$.That is to say Z_μ has three components, considering the spin of gauge field ($J = 1$), is considered the interaction between the spin, the different of three components of gauge field $A_\mu(\vec{r},t)$ is the spin interaction. In a hydrogen atom, for example, we illustrate the relationship between the g_s and e. As we know, due to electromagnetic interactions, the hydrogen energy level splits about $1eV$, and the energy level splitting due to the interaction between electron spin and nuclear spin (so-called hyperfine structure) is about $10^4 MHz$[49, 50], so the ratio of energy splitting between the electromagnetic interaction with spin interaction is

$$\frac{\triangle E_s}{\triangle E_e} \sim 10^{-6}. \tag{134}$$

Considering $\triangle E_e \sim e^2, \triangle E_s \sim g_S^2$,there is

$$\frac{g_S}{e} \sim 10^{-3}. \tag{135}$$

B. Interactions of $A(T_d)$ particles :flavor,color interactions

This family of particles with group symmetry $S(T_d)$, the algebra of their generators is $A(T_d)$, it can be decomposed into direct sum of several subalgebraes (see Formula (19)).We rewrite it below.

$$A(T_d) = \sum_{i=1}^{5} \oplus A_0^{id} \oplus A_1^S \oplus A_2^F \oplus A_2'^C \cong A(O). \tag{136}$$

The superscript S, F, C respectively are corresponding to spin, flavor and color. With similar (uds) of quarks, A_1^S will lead to spin interaction (same as $SU_S(2)$ but different),A_2^F will lead to flavor ($l_{\nu L}, l_L, l^c_L$) interaction (same as $SU_F(3)$ but different), $A_2'^C$ will lead to color($e\mu\tau$) interaction (same as $SU_C(3)$ but different).By analysis of spin interaction in Section V(A), if combining spin and flavor or color state and become Dirac particles with spin 1/2, the interaction between the spin can be combined to study in the interaction between flavor (or color) where the gauge field with spin 1.To this end, will not be discussed in the following discussion of the interaction between the spin independently.

1. Flavor(spin) dynamics.

The basis of flavor A_2^F (including spin) is left-handed state (see Formula (50)),

$$\Psi_F = \begin{pmatrix} l_{\nu L} \\ l_L \\ l_L^c \end{pmatrix} = \begin{pmatrix} l_\nu \\ l \\ l^c \end{pmatrix}_L . \tag{137}$$

Because of the symmetry $S(T_d)$ of spacetime, when only consider flavor (spin), the gauge transformation of particle state Ψ_F is

$$\Psi_F(\vec{r},t) = \exp[-i\omega^\gamma(\vec{r},t)\tau_\gamma - i\omega^i(\vec{r},t)T_i]_F(\vec{r},t). \tag{138}$$

Where $\tau_\gamma \in A_1^S, T_i \in A_2^F$.Because of gauge invariance, the covariant derivative

$$D_\mu = \partial_\mu - ig_F A_\mu^i(\vec{r},t)T_i. \tag{139}$$

So The Lagrangian of interaction with gauge field is

$$\pounds = i\overline{\Psi}_F\gamma_\mu\partial_\mu\Psi_F + \frac{g_F}{\sqrt{2}}\overline{\Psi}_F\gamma_\mu A_\mu(\vec{r},t)\Psi_F - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{140}$$

here

$$F_{\mu\nu} = (\partial_\mu A_\nu^i - \partial_\nu A_\mu^i)T_i + g_F f_{ij}^k A_\mu^i A_\nu^j T_k, \tag{141}$$

$$A\mu(\vec{r},t) = \begin{pmatrix} \frac{A_\mu^3}{\sqrt{2}} + \frac{A_\mu^8}{\sqrt{6}} & \overline{W}_\mu^1 & \overline{W}_\mu^2 \\ W_\mu^1 & -\frac{A_\mu^3}{\sqrt{2}} + \frac{A_\mu^8}{\sqrt{6}} & \overline{W}_\mu^3 \\ W_\mu^2 & W_\mu^3 & -2\frac{A_\mu^8}{\sqrt{6}} \end{pmatrix}, \tag{142}$$

where

$$W_\mu^1 = \frac{A_\mu^1 + iA_\mu^2}{\sqrt{2}}, W_\mu^2 = \frac{A_\mu^4 + iA_\mu^5}{\sqrt{2}}, W_\mu^3 = \frac{A_\mu^6 + iA_\mu^7}{\sqrt{2}}. \tag{143}$$

From Formula (140) we get interaction term of particles and gauge fields below,

$$\pounds_{fg} = \frac{g_F}{\sqrt{2}} \begin{pmatrix} l_{\nu L} & l_L & l_L^c \end{pmatrix} \gamma_\mu A\mu(\vec{r},t) \begin{pmatrix} l_{\nu L} \\ l_L \\ l_L^c \end{pmatrix}. \tag{144}$$

In view[51, 52] of $l_L^c = c\bar{l}_R^T, l_R^c = c\bar{l}_L^T$ (where c is constant operator and $c = i\gamma_2\gamma_4$ in Dirac representation)there are equation about some terms of gauge field component following,

$$\bar{l}_{\nu L}\gamma_\mu\overline{W}_\mu^2 l_L^c = 0, \tag{145}$$

$$\bar{l}_L^c\gamma_\mu\overline{W}_\mu^3 l_L^c = 0, \tag{146}$$

$$\bar{l}_L^c\gamma_\mu\overline{W}_\mu^2 l_{\nu L} = 0, \tag{147}$$

$$\bar{l}_L^c\gamma_\mu\overline{W}_\mu^3 l_L = 0, \tag{148}$$

$$\bar{l}_L^c\gamma_\mu A_\mu^8 l_L^c = -\bar{l}_R\gamma_\mu A_\mu^8 l_R. \tag{149}$$

So Formula (144) becomes

$$\pounds_{fg} = \frac{g_F}{\sqrt{2}} \begin{pmatrix} \bar{l}_\nu & \bar{l} \end{pmatrix} \gamma\mu \begin{pmatrix} \frac{1+\gamma_5}{2}(\frac{A_\mu^3}{\sqrt{2}} + \frac{A_\mu^8}{\sqrt{6}}) & \frac{1+\gamma_5}{2}\overline{W}_\mu^1 \\ \frac{1+\gamma_5}{2}W_\mu^1 & \frac{1+\gamma_5}{2}(-\frac{A_\mu^3}{\sqrt{2}}) + \frac{3-\gamma_5}{2}\frac{A_\mu^8}{\sqrt{6}}) \end{pmatrix} \begin{pmatrix} l_\nu \\ l \end{pmatrix}. \tag{150}$$

It is a theory with two components(l_ν, l) in which can compare with the Standard Model. This interaction showed that followings. (1) The charge current is pure V-A. (2) The neutral current between neutrinos is also pure V-A. (3) The neutral current between leptons is also V-A, but not pure V-A neutral current. Qualitative, above results is full compliance with existing experiment about lepton. And more convincing quantitative results are the following.Let

$$\begin{aligned} \alpha Z\mu &= \tfrac{1}{\sqrt{2}}A_\mu^3 + \tfrac{1}{\sqrt{6}}A_\mu^8, \\ A_\mu &= aA_\mu^3 + bA_\mu^8. \end{aligned} \tag{151}$$

Where α, a, b is constant to be confirmed. Put Formula (151) into Formula (150),the interaction term can be rewritten as

$$\pounds_{fg} = \frac{g_F}{\sqrt{2}} \begin{pmatrix} \bar{l}_\nu & \bar{l} \end{pmatrix} \gamma\mu \begin{pmatrix} \frac{1+\gamma_5}{2}\alpha Z\mu & \frac{1+\gamma_5}{2}W_\mu^+ \\ \frac{1+\gamma_5}{2}W_\mu^- & \frac{\sqrt{2}}{\sqrt{3}b-a}A_\mu + (\frac{\frac{1}{2}b+\frac{\sqrt{3}}{2}a}{\frac{1}{\sqrt{3}}a-b} - \frac{\gamma_5}{2})\alpha Z_\mu \end{pmatrix} \begin{pmatrix} l_\nu \\ l \end{pmatrix}. \tag{152}$$

Where $W_\mu^- \equiv W_\mu^1, W_\mu^+ \equiv \overline{W}_\mu^1$.According to compare with GWS modelthe Lagrangian of interaction in the G-WS model for leptons is

$$\pounds_{fg}^{GWS} = \frac{g_F}{\sqrt{2}} \begin{pmatrix} \bar{l}_\nu & \bar{l} \end{pmatrix} \gamma\mu \begin{pmatrix} \frac{1+\gamma_5}{2}\rho Z\mu & \frac{1+\gamma_5}{2}W_\mu^+ \\ \frac{1+\gamma_5}{2}W_\mu^- & \chi A_\mu + \rho(4\sin^2\theta_W - 1 - \gamma_5)Z_\mu \end{pmatrix} \begin{pmatrix} l_\nu \\ l \end{pmatrix}. \tag{153}$$

Where $g\sin\theta_W = g'\cos\theta_W = e, \theta_W$ is Weinberg angle of GWS model,and

$$\begin{cases} \rho = \frac{\sqrt{2(g^2+g'^2)}}{4g}, \\ \chi = -\frac{\sqrt{2}g'}{\sqrt{2(g^2+g'^2)}}. \end{cases} \tag{154}$$

If Formula (153) and (152) are same theory, we have following relations.

$$\begin{cases} \alpha\frac{g_F}{2\sqrt{2}} = \frac{\sqrt{2(g^2+g'^2)}}{4g}, \\ g_F = g, \\ \frac{g_F}{\sqrt{3}b-a} = -e, \\ \frac{\frac{1}{\sqrt{2}}\alpha g_F(b+\sqrt{3}a)}{\frac{1}{\sqrt{3}}a-b} = \frac{\sqrt{g^2+g'^2}}{2}(4\sin^2\theta_W - 1). \end{cases} \tag{155}$$

If formula (151) is regular rotation, we have following relations.

$$\begin{cases} \alpha = \sqrt{\frac{2}{3}}, \\ a = \frac{1}{2}, \\ b = -\frac{\sqrt{3}}{2}. \end{cases} \tag{156}$$

According to relation Formula (155) we can get

$$\begin{cases} \sin^2\theta_W = \frac{1}{4}, \\ g_F = 2e = \sqrt{\frac{16\pi}{137}}. \end{cases} \tag{157}$$

As Higgs field of quarks (uds), (see Formula (111)∼(119)), easy to get mass of gauge particles $W^{\pm}, Z^0$

$$m_{W^{\pm}} = \alpha m_{z^0} = \frac{1}{\sqrt{2}} g_F l_0. \tag{158}$$

According to the experiment value of group UA1 in 1983[53],

$$m_{W^{\pm}} = 80.9 \pm 1.5 \pm 3.0 GeV. \tag{159}$$

We can accurately calculate l_0,

$$l_0 = (1.89 \pm 0.08) \times 10^2 GeV. \tag{160}$$

It is equivalent to length scale

$$l_0 = \frac{hc}{E_{l_0}} = 0.957 \times 10^{-18} m. \tag{161}$$

From a simple consideration that transformation Formula (151) is regular rotation, we theoretically get the Weinberg Angle $\theta_W = \frac{1}{6}\pi$,and get the mass ratio $\alpha = \sqrt{\frac{2}{3}}$ of $W^{\pm}$ and Z^0.The result meet quite well with the experiment, it showes my theory of rationality.

2. *Color(spin) dynamics*

The particles family have three colors $(e\mu\tau)$, the basis (including spin) of color Lie algebra A_2^C are the left-hand state (see Formula (50)),

$$\Psi_C = \begin{pmatrix} l_\nu \\ l \\ l^c \end{pmatrix}_L . \tag{162}$$

Because of the symmetry $S(T_d)$ of spacetime, when only consider color(spin), the gauge transformation of particle state Ψ_C is

$$\Psi'_C(\vec{r},t) = \exp[-i\omega^\gamma(\vec{r},t)\tau_\gamma - i\omega^i(\vec{r},t)T_i]\Psi_C(\vec{r},t). \tag{163}$$

Where $\tau_\gamma \in A_1^S, T_i \in A_2^C$.Because of gauge invariance, the covariant derivative

$$D_\mu = \partial_\mu - ig_C A_\mu^i(\vec{r},t)T_i. \tag{164}$$

So The Lagrangian of interaction with gauge field is

$$\pounds_{fg} = \frac{g_C}{\sqrt{2}} \begin{pmatrix} \bar{e} & \bar{\mu} & \bar{\tau} \end{pmatrix}_L \gamma_\mu \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & -(G_{11}+G_{22}) \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L . \tag{165}$$

Where g_C is interaction coupling constant of color $(e\mu\tau)_L$.

$$G_{11} = \frac{A_\mu^3}{\sqrt{2}} + \frac{A_\mu^8}{\sqrt{6}}, \quad G_{12} = \frac{A_\mu^1 - iA_\mu^2}{\sqrt{2}}, \quad G_{13} = \frac{A_\mu^4 - iA_\mu^5}{\sqrt{2}},$$
$$G_{21} = \frac{A_\mu^1 + iA_\mu^2}{\sqrt{2}}, \quad G_{22} = -\frac{A_\mu^3}{\sqrt{2}} + \frac{A_\mu^8}{\sqrt{6}}, \quad G_{23} = \frac{A_\mu^6 - iA_\mu^7}{\sqrt{2}},$$
$$G_{31} = \frac{A_\mu^4 + iA_\mu^5}{2}, \quad G_{32} = \frac{A_\mu^6 + iA_\mu^7}{2}.$$

There are the eight vector field with spin 1,is called as gluon field. It is similar to the interaction of quarks (uds) with colors (RBG), and not same in the color $(e\mu\tau)$.

The color interaction Formula (165) tells us that the interaction between the colors $(e\mu\tau)$ is pure V-A type. It indicates that under the interaction of color, the color can mutual transformation between $(e\mu\tau)$, i.e., lepton number is no longer conserved. But this may be on the higher energy level.

Now we show again the rationality of my theory through the ratio R of electron-positron collision. By the definition of R

$$R = \frac{\sum e\bar{e} \to q\bar{q}(hadron)}{e\bar{e} \to \mu\bar{\mu}} = \sum_q e_q^2. \tag{166}$$

In the usual quark theory, hadron here include (uds) and quarks (cbt) with charge $(2/3, -1/3, 2/3)$.And in my theory, quarks (cbt) are the lepton with different charge $(0, -1, 1)$.There are the several kinds of Feynman diagrams of electron-positron. Therefore,

(1) when the energy of $(e\bar{e})[i.e., l^e\bar{l}^e]$ is enough to produce (uds), $E < 3100MeV$,

$$R = \frac{3[(\frac{2}{3})^2 + (-\frac{1}{3})^2 + (-\frac{1}{3})^2]}{1} = 2.$$

(2) When the energy of $(e\bar{e})$ is enough to produce (uds) and particle $J/\Psi(l^e\bar{l}^e), 3100MeV < E < 9460MeV$,

$$R = \frac{3[(\frac{2}{3})^2 + (-\frac{1}{3})^2 + (-\frac{1}{3})^2] + [0^2 + 1^2 + (-1)^2]}{1} = 4.$$

(3) When the energy of $(e\bar{e})$ is enough to produce (uds) and particle J/Ψ,particle $Y(l^\mu\bar{l}^\mu), 9460MeV < E < Ex(?)$,

$$R = \frac{3[(\frac{2}{3})^2 + (-\frac{1}{3})^2 + (-\frac{1}{3})^2] + 2[0^2 + 1^2 + (-1)^2]}{1} = 6.$$

(4) When the energy of $(e\bar{e})$ is enough to produce (uds) and particle J/Ψ,particle Y,particle $X(l^\tau\bar{l}^\tau), E > Ex(?)$,

$$R = \frac{3[(\frac{2}{3})^2 + (-\frac{1}{3})^2 + (-\frac{1}{3})^2] + 3[0^2 + 1^2 + (-1)^2]}{1} = 8.$$

Draw a diagram in above value of R in Figure X.

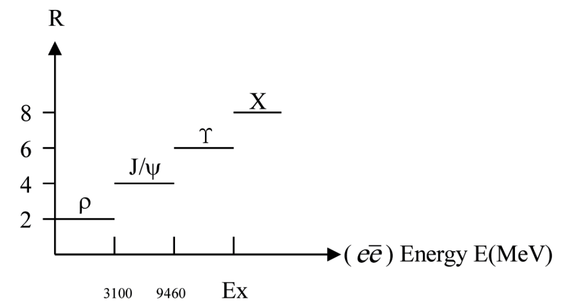

Figure 10 Relation of R−E($e\bar{e}$)

The above estimates meet the experiment better[40].And is better than the theory of usual quark. So far, I have established the unificational interaction theory determined by the spacetime structure in addition to the gravity. In the next section, I will be devoted to gravitational interaction. After gravitational theory, we will see, gravitational interactions with other three kinds of interaction (flavor, color, spin) are unified in the symmetry of the structure of spacetime.

VI. GRAVITY

Einstein's general relativity is the most perfect gravitational interaction theory so far. It is proved by a series of experiments, for example, mercury precession[54].It has a very beautiful mathematical form[55].

In order to compare my gravity theory with Einstein's general relativity, I will first give the pure geometry form of Einstein's general relativity in the four-dimensional continuum spacetime, and points out its shortcomings.

A. Introduction

In the case of matter L, as in the form of gravitational action

$$S = \int_M d^4x\sqrt{-g}(R + L). \tag{167}$$

The symbols are the usual meanings. Here would like to point out. (1) In the material field (Lagrangian density)L, because this part is not a geometric, not spacetime geometric interpretation in time. The spacetime theory is artificial and determined by the artificial matter. In the geometry, it is not very natural. (2) Generally the $g_{\mu\nu}$ is recognized as the gravitational field, and will $\Gamma^\lambda_{\mu\nu}$ as a functional of $g_{\mu\nu}$, their relationship is:

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\tau}(g_{\mu\tau,\nu} + g_{\nu\tau,\mu} - g_{\mu\nu,\tau}). \tag{168}$$

This is not satisfactory in geometry. I will use the geometric method to analyses the geometric meaning of all the symbols of Einstein's theory of gravity Formula (167).We will point out that which kind of quantity should be used and make new answer to the above two.

B. Description of the spacetime

1,Manifold M, in general gravity problem, M can take into R^4,can also be $R^3 \times S$, but the equivalence principle tells us that locally M should be R^4.From the viewpoint of description for any spacetime, M can be any a manifold. In the following discussion, we think M is an arbitrary manifold.

2,Given M, we can naturally define a family of function F, which makes the mappings $f : M \to R^1$ is a continuous function of any order $(C^\infty), f \in F$ here.

3,Given M (naturally have got F), there are an infinite number of mappings $v : f \to R^1$ at any point q on the M. The mappings are the tangent vector; they form a tangent vector space. There is a tangent vector space V_q at each q on M. The tangent vector space V_q and $V_{q'}$ at the difference point q, q' on M,their dimensions are equal, but are different, which there is no connection between these two vectors. If specified a tangent vector at every point on the M, then these vectors form a tangent vector field on the M.

4,Given M (naturally got $F, V_q, q \in M$), there are always many linear mappings $V^*_q : V_q \to R^1$.A collection of these linear mapping constitutes a linear space - dual space V^*_q at the point q.Therefore, given M, there is a dual space V^*_q naturally at every point on the M. But, there is no corresponding relationship between the space V_q and dual space V^*_q at the same point q. Forms a dual vector field by specifying a dual space vector for every point on the M.

5,Given M (naturally got F,$V_q, V^*_q, q \in M$), we can define a multiple linear mapping at point q on the M,

$$T : \underbrace{V^* \times V^* \times \times \cdots \times V^*}_{k} \times \underbrace{V \times V \times \cdots \times V}_{l} \to R^1. \tag{169}$$

All of these mappings constitute a $T(k, l)$ tensor space at the q on the M. For each type of map (tensor) we specified a map at every point on the M, these tensors form a type $T(k, l)$ tensor field.

6,Description of spacetime. Above all is the mathematical analysis, no physical content. So, what is a physical spacetime? That's to appoint one on the mathematically variable, and a set of the comparative method at difference point.

The first is to appoint a manifold. Einstein and generations not much has explained this, for example, what is given? We will give a very good answer in the two behind.

After a given M, we will naturally have a vector space V_q and dual space V^*_q at each point q on the manifold M.

If given one-to-one correspondence relationship between the vectors on V_q and V_q^*,namely mapping

$$g : V_q \to V_q^*, (q \in M), \tag{170}$$

then the inner product definition is given on the M, and the definition of an infinitesimal displacement size is given (i.e., the distance between any two adjacent points)

$$ds^2 = g_{ab}V^aV^b = V_aV^b, (V_a, V_b \in V^*; V^a, V^b \in V). \tag{171}$$

Next, there is this inner product definition (Formula (171)) at each point on M, but there is no relationship of the inner product at different point on the M. If given relationship of the inner product at different point on the M, we can compare inner product at different point. It is just the function of the connection Γ^c_{ab}.

Know the $(M, g_{ab}, \Gamma^c_{ab})$, any tensor can be described on the manifold M. That is to say, spacetime can described completely by $(M, g_{ab}, \Gamma^c_{ab})$.This also means that the action of spacetime only related to the three $(M, g_{ab}, \Gamma^c_{ab})$, and has nothing to do with the other. However, according to the current Einstein gravitational theory, besides these three, and material field (see Formula (167)), namely in advance to specify a tensor field on the M. This description of spacetime is not acceptable within mathematics framework. However, experiments show spacetime geometry is determined by material field, so what is the material? Material fields how to influence spacetime geometry? Because,g_{ab},Γ^c_{ab} is a kind of physical specified, it is depends on the material field, it can't load material field. The only possible is the manifold M to load material field. The next analysis also proves it is just M.

C. Pure geometric gravity action

Besides non-geometrizational material field in Formula (167), contained the material field to the structure of the manifold M, the geometry action of gravity is

$$S = \int_M d^4x\sqrt{-g}R. \tag{172}$$

R is the scalar curvature of spacetime. We see only one point particle. On the M, if there is not this particle, $M = R^4$. After joining this point particle, the manifold can be approximate the rest as remove this point from $R^4, M = R^4 - P$.So,in this spacetime with one point particle p, gravity action of Formula (172) is

$$S = \int_{R^4-p} d^4x\sqrt{-g}R. \tag{173}$$

Formula (173) means that the scalar curvature R is defined in R^4 except point p. For convenient to calculate, we also define a number R_p in point p, and the total integral interval expand to the whole R^4, it should be subtract the expanding part of the integrand in the action, namely

$$S = \int_{R^4} d^4x\sqrt{-g}(R - c\delta(p)). \tag{174}$$

Where c is a constant to describe the properties of particles and to affect R. The readers who are familiar with general relativity can see immediately that the solution of Formula (174) is outside solution of the Schwarzschild solution[54].

If denoted $-c\delta(P)$ in Formula (174) by

$$L = -c\delta(p), \tag{175}$$

the pure geometric gravity Formula (167) becomes gravity action with a material field L, in the situation, $M = R^4$ of this time. For multiple particle situations, similar result can give in Formula (174),

$$S = \int_{R^4} d^4x\sqrt{-g}(R + L). \tag{176}$$

Formula (176) is equivalent to Einstein's general theory of relativity with a material field. This shows that our consideration for Formula (172) is appropriate, namely material field is the structure of the manifold M itself, is in line with my front theory conclusion. Let me to see how the gravitational field depends on the material structure(spacetime structure (M)).

D. Gravitational field in matter spacetime

In the previous sections, I established a new spacetime structure, that is: the vacuum manifold is $\mathcal{A}_1 \times T$, and matter spacetime manifold is $\mathcal{A}_1 \times T$ with deformation,a hole or gap cell or cell deformation, T is the time lattice. The two manifolds are discrete manifold;all cannot use continuous method to deal with.In this kind of manifold, however,has a nature vector structure of position vector:let $p \subset \mathcal{A}_1 \times T$ as the origin,to all around the center of the cell,has a structure composed of an infinite number vectors.In a vacuum spacetime,these infinite vectors satisfy O_h or T_d symmetry in cell lattice face centered cubic $\mathcal{A}_1$,and in the matter spaceime,the structure of infinite vectors satisfy the symmetry of $S(O_h)$ or $S(T_d)$ $(= R(\gamma^i_\alpha)T_\alpha)$ in cell lattice face centered cubic $\mathcal{A}_1$ with deformation.The vector structure has different forms in the different matter spacetime, while the matter spacetime and particle state is associated, so the particle state is actually with a center of hole or gap cell as the origin of a special vector structure. In the matter spacetime, because of the symmetry O_h or T_d,only some special vector structure is possible, this special particle vector structure is different particle state. If to introduce continuous tangent vector space at the point p, so, this is equivalent to specify some discrete vector in the continuous space vector structure and this specific vector structure is as particle state. This suggests that, in my theory, if there is a continuous tangent vector space at point p, particle state at point p is not use a vector to describe, but is to specify a batch of tangent vector to describe (unlimited number of vectors).

In a vacuum spacetime,the vector structure at the every point p in the $\mathcal{A}_1 \times T$ are the same. In the matter spacetime, the vector structure at every point p is different in principle. To compare the difference vector structure must be to introduce the concept of connection.

In the past theory, the tangent vector field at each point p to specify one vector. With the language of fiber bundle theory, the tangent vector field is equivalent to a cross section in which every point be specified by one point on the tangent bundle fiber $T_p(M)$ at point p on M, as shown in Figure 11.

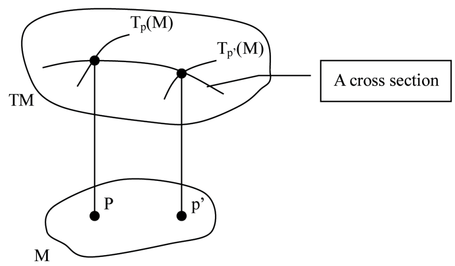

Figure 11 The cross section on the tangent bundle fiber

In my theory, particle state, the tangent vector structure, are so specified in the fiber on $T_p(M)$ is not a point, but an unlimited number of discrete points. In this way, the particle is no longer just a cross section, but a group of the cross section, as shown in Figure 12.

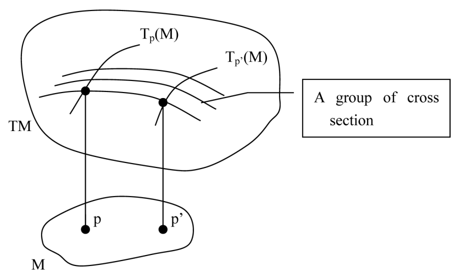

Figure 12 The group of cross section on the tangent bundle fiber

From the above discussion caused us to describe physical separation in continuous situation. Method is as follows: $\mathcal{A}_1 \times T$ can be seen as a set of some discrete points in 4-dimension smooth manifold R^4.Means: (1) each discrete point p belong to $\mathcal{A}_1 \times T$ are naturally have a four dimensional tangent vector space, the vector structure with origin point p can be described by using the tangent vector space.(2) On the $\mathcal{A}_1 \times T$ a trajectory should be some of the discrete point and no line. But we can use a smooth curve approximation (C^∞) on the smooth manifold R^4, makes the discrete points to be the discrete points of the curve. So, my theory can use the continuous method to describe. In the continuous case, fibers of the point p and p' are $T_p(M)$ and $T'_p(M)$ respectively. The symmetry of $T_p(M)$ and $T'_p(M)$ are $GL(m,R)$.The affine connection, the connection on the tangent bundle, is entirely determined by material fields $A^\alpha_\mu(\vec{r},t)$ [56],

$$\Gamma^\nu_{\mu k} = (A^\alpha_\mu \frac{\lambda^\alpha}{2i})^\nu_k, \tag{177}$$

where λ^α is the generators of $GL(m,R)$.

In my model, the symmetry of fiber $T_p(M)$ and $T'_p(M)$ are also $GL(m,R)$.Physically meaningful, however, is not this symmetry, but is the symmetry of infinite discrete point specified on $T_p(M)$ and $T'_p(M)$,i.e., the symmetry of vector structure ant p and p'. In principle, in the matter spacetime, the vector structure symmetry at the difference p and p' are different.

Let M is 4-dimension smooth manifold, with a coordinate system $(U;\mathbf{u}^i)$, has the natural basis $\{s_i = \frac{\partial}{\partial u_i}, 1 \le i \le 4\}$,which constitute the local frame field of a tangent bundle $T(M)$ on U.Given a connection D on tangent bundle[57],

$$Ds_i = \omega^j_i \otimes s_j = \Gamma^j_{ik} du^k \otimes s_j, \tag{178}$$

where we do not limit the connection in M and it is given later.In affine space $T_p(M)$ whose connection do not given, for each point p on M, its properties can completely be described by this tangent vector space(with infinite number),which is a set of vectors from p to every center of cell,$\{\vec{r}_{pn}, n = 0,1,2,...\}$,and such vector $\vec{r}_{pn}$ can be represented completely by natural base $\{s_i\}$,

$$\hat{r}_{(pn)i} = X^j_{(pn)i}(\frac{\partial}{\partial u^j})_p, (i,j = 1,2,3,4), \tag{179}$$

which $X^j_{(pn)i}$ is a 4×4 non-degeneration matrix, with different origin p, has different value to different vector n, fully determined by the structure of $\mathcal{A}_1 \times T$. At the same time, it is also an element of $GL(4,R)$.The $\{\hat{r}_{(pn)i}\}$ is essentially many discrete point of the fiber $T_p(M)$ on the tangent bundle $T(M)$, $(p, \hat{r}_{(pn)})$ can constitute section group under certain conditions. Physically, the sectional set parallel to each other (i.e.,no two cells in same position),require namely

$$D\hat{r}_{(pn)i} = 0. \tag{180}$$

According to Formula (178) and (179), Formula (180) becomes

$$\frac{dX^k_{(pn)i}}{du^l} + X^j_{(pn)i}\Gamma^k_{jl} = 0. \tag{181}$$

Because $X^j_{(pn)i}$ is completely determined by the structure of $\mathcal{A}_1 \times T$ with deformation, such a deformation

$X^{j}_{(pn)i}$ of certain material field (or vacuum) is known, then the connection can be obtained in Formula (181),

$$\Gamma^{k}_{j'l} = -X^{*i}_{(pn)j'} \frac{dX^{k}_{(pn)i}}{du^{l}}, \tag{182}$$

where

$$X^{*i}_{(pn)j'} X^{j}_{(pn)i} = \delta^{j'}_{j}. \tag{183}$$

Formula (182) means that the connection of matter s-pacetime is determined completely by the structure of the matter spacetime. That is to say that the gravitational field is completely determined by material field.

(a) In vacuum spacetime,$X^{k}_{(pn)i}$ is a constant matrix,

$$\frac{dX^{k}_{(pn)i}}{du^{l}} = 0 \rightarrow \Gamma^{k}_{jl} = 0. \tag{184}$$

It means that the vacuum is flat.

(b)In the matter spacetime,$X^{j}_{(pn)i}$ are deviation of $(X^{j}_{(pn)i})_{vacuum} \equiv (X^{j}_{(pn)i})_{v}$ in a vacuum, this deviation is related with the coordinates, denoted by

$$[\Psi_{(pn)}(\mathbf{u}^{l})]^{j}_{i}, \tag{185}$$

namely

$$X^{k}_{(pn)i} = [\Psi_{(pn)}(\mathbf{u}^{l}) X_{(pn)v}]^{k}_{i}. \tag{186}$$

So

$$\Gamma^{k}_{j'l} = -[\Psi^{*}_{(pn)}(\mathbf{u}^{l'})]^{i}_{m} [X^{*}_{(pn)v}]^{m}_{j'} \frac{d[\Psi_{(pn)}(\mathbf{u}^{l'})]^{k}_{k'}}{d\mathbf{u}^{l}} [X_{(pn)v}]^{k'}_{i}. \tag{187}$$

In the matter spacetime,

$$\frac{d[\Psi_{(pn)}(\mathbf{u}^{l'})]^{k}_{k'}}{d\mathbf{u}^{l}} \neq 0. \tag{188}$$

It indicates that $\Gamma^{k}_{j'l}$ must be not zero. Considering the gravitational field, the matter spacetime must be curved, and can be calculated by the gravitational field in Formula (187).

The above two conclusions on qualitative is consistent with Einstein's theory. If we can know the structure of vectors $\{\hat{r}_{(pn)}\}$ corresponding to the structure of particles, we can calculate the gravitational field in Formula (187) on different particle state. This is possible in principle.

Formula (182) clearly shows that the gravitational field is source of the translation characteristics of spacetime structure. This completes a dream that the basic interaction will be unified in structure of spacetime.

VII. SUMMARIZATION

By simple ideal to give a structure of spacetime, such a structure is that the vacuum space is the structure of cell with diameter l_0 in the face-centered cubic close structure $\mathcal{A}_1$.Then we define the matter space, vacuum and particle. We study detail the symmetry of vacuum space and the matter space. By the definition of the time, we got a whole series of conceptions of inertial system, speed, and lattice wave in the discrete spacetime. By vacuum translation invariance and light being the lattice wave,we got relativity theory as Einstein's special relativity theory same as. Through detailed analysis of symmetry of matter spacetime, we obtained the decisive local gauge invariance Formula (16), and the generators of this gauge invariance is elements of point group O_h or T_d, the algebra of the generators is group algebra $A(O_h)$ for O_h and group algebra $A(T_d)$ for T_d. By the integrity study of $A(O_h)$ and $A(T_d)$,we found that they are the direct sum of some Lie algebra. Thus the particles (i.e., representation of the generators) is divided into the representation of $A(O_h)$ and $A(T_d)$,and gauge particles corresponding to the generators of local gauge invariance. Exciting is that this package arrangement was successful: the group algebra $A(O_h)$ constructed all Gell-mann quark (uds),the group algebra $A(T_d)$ constructed all leptons and so-called quark (cbt) particle, the local gauge theory constructed the interactions between spins,flavors,colors,all results are consistent with the experiment.For example,the chirality of interaction, the structure of currents, the structure of gauge field, the value of R,etc. It's particularly worthy to point out that in my interaction between lepton flavor $(l_{\nu L} l_L l^c_L)$ we can calculate the Weinberg Angle as $\theta_W = 30^o$,and its precision consistent with the experimental results, and intuitive interpretation of the Higgs field.These results prove the rationality of my theory. Through the mass of W, Z^0 in experiments, we get the scale of the vacuum $l_0 \sim 10^{-18}m$,in line with the existing theoretical estimates. Through the analysis of the Einstein gravitational theory, puts forward the basic concept of the pure geometric gravitational field are three:manifold M, metric g_{ab} and connection Γ^{c}_{ab}.The material field is the manifold, that is coincide with the my spacetime theory.Through the analysis of spacetime symmetry, we obtained the relation Formula(182) which between the gravitational field is determined by the material field.It shows that the gravitational field is the result of translational symmetry of the matter spacetime.It makes us to unify the gravitational interaction,the strong interaction ,electroweak interaction and all particles in the structure of spacetime.The point group symmetry of s-pacetime lead to strong, electroweak interactions and all particles, the translational symmetry of spacetime lead to gravitational interaction.

VIII. APPENDIX GROUP ALGEBRA OF T_d AND O_h

A. Group T_d and O_h

Group T_d is complete symmetry group of regular tetrahedron,and is type P extrinsic point group of group O(excluding space inversion).Group O_h is complete symmetry group of octahedron,and is type I extrinsic point group of group O (including space inversion).Because $O_h = O \times I$,group T_d and group O are math isomorphism, so we study group O first.

TABLE XI. Corresponds between O,T_d and S_4

Symbol	Element of S_4	Element of T_d	Element of O	Classes
$T_0 = E$	E	E	E	E
T_1	(12)(34)	$C_4(001)$	$C_4(001)$	$C_4^2(3)$
T_2	(13)(24)	$C_4(100)$	$C_4(100)$	
T_3	(12)(34)	$C_4(010)$	$C_4(010)$	
T_4	(123)	$C_3(1-11)$	$C_3(1-11)$	$C_3'(8)$
T_5	(142)	$C_3(-111)$	$C_3(-111)$	
T_6	(134)	$C_3(11-1)$	$C_3(11-1)$	
T_7	(243)	$C_3(111)$	$C_3(111)$	
T_8	(132)	$C_3(1-11)$	$C_3(1-11)$	
T_9	(124)	$C_3(-111)$	$C_3(-111)$	
T_{10}	(143)	$C_3(11-1)$	$C_3(11-1)$	
T_{11}	(234)	$C_3(111)$	$C_3(111)$	
T_{12}	(12)	$C_2(110)\sigma$	$C_2(110)$	$C_2''(6)$
T_{13}	(13)	$C_2(011)\sigma$	$C_2(011)$	
T_{14}	(14)	$C_2(101)\sigma$	$C_2(101)$	
T_{15}	(23)	$C_2(-101)\sigma$	$C_2(-101)$	
T_{16}	(24)	$C_2(01-1)\sigma$	$C_2(01-1)$	
T_{17}	(34)	$C_2(1-10)\sigma$	$C_2(1-10)$	
T_{18}	(1234)	$C_4(100)\sigma$	$C_4(100)$	$C_4(6)$
T_{19}	(1243)	$C_4(010)\sigma$	$C_4(010)$	
T_{20}	(1324)	$C_4(001)\sigma$	$C_4(001)$	
T_{21}	(1432)	$C_4(100)\sigma$	$C_4(100)$	
T_{22}	(1342)	$C_4(010)\sigma$	$C_4(010)$	
T_{23}	(1423)	$C_4(001)\sigma$	$C_4(001)$	

Group O is isomorphic to permutation group S_4, it-s order is 24, the relation of their group element shown in Table XI.From the multiplication table of S_4 we can immediately get multiplication Table 14 of group O.We can be seen that the set $E, T_1, T_2, ..., T_{11}$ these 12 elements constitute a normal subgroup of group O from Table 14,it is group T.

B. Group algebra $A(O)$ of group O

Let $A(O)$ denote group algebra of group O.By Tables 14 it's easy to see $A(O)$ is Lie algebra[58].(in fact, group algebra of almost all finite group are Lie algebra).Because of limited dimension of $A(O)$, it must be the direct sum of some known semi-simple Lie algebras.

From Table 14,we can know,

(1)(T_1, T_2, T_3) commutate each other;
(2)T_1 and $(T_{12}, T_{17}, T_{20}, T_{23})$ commutate;
T_2 and $(T_{13}, T_{16}, T_{18}, T_{21})$ commutate;
T_3 and $(T_{14}, T_{15}, T_{19}, T_{22})$ commutate.
Order

$$\begin{cases} H_1 = T_1, H_1^{'} = T_{12} + T_{17} - T_{20} - T_{23}, \\ H_2 = T_2, H_2^{'} = T_{13} + T_{16} - T_{18} - T_{21}, \\ H_3 = T_3, H_3^{'} = T_{14} + T_{15} - T_{19} - T_{22}. \end{cases} \tag{189}$$

We have

$$[H_i, H_j] = 0, [H_i, H_j^{'}] = 0, [H_i^{'}, H_j^{'}] = 0, (i, j = 1, 2, 3). \tag{190}$$

They can possible to construct a Carton subalgebra. Order

$$\begin{cases} E_{\alpha} = T_4 + T_5 - T_6 - T_7, \\ E_{-\alpha} = T_8 + T_9 - T_{10} - T_{11}, \\ E_{\beta} = T_4 - T_5 - T_6 + T_7, \\ E_{-\beta} = T_8 - T_9 - T_{10} + T_{11}, \\ E_{-(\alpha+\beta)} = T_4 - T_5 + T_6 - T_7, \\ E_{(\alpha+\beta)} = T_8 - T_9 + T_{10} - T_{11}; \\ \\ E_{\alpha}' = T_{12} - T_{17} - T_{20} + T_{23}, \\ E_{-\alpha}' = T_{12} - T_{17} + T_{20} - T_{23}, \\ E_{\beta}' = T_{14} - T_{15} - T_{19} + T_{22}, \\ E_{-\beta}' = T_{14} - T_{15} + T_{19} - T_{22}, \\ E_{-(\alpha+\beta)}' = T_{13} - T_{16} + T_{18} - T_{21}, \\ E_{(\alpha+\beta)}' = T_{13} - T_{16} - T_{18} + T_{21}. \end{cases} \tag{191}$$

We have their commutation results shown in Table XII.From Table XII. we know:

(1)$(H_i, E_{\pm\alpha}, E_{\pm\beta}, E_{\pm(\alpha+\beta)})$ constitute a Lie algebra A_2,

(2)$(H_i', E_{\pm\alpha}', E_{\pm\beta}', E_{\pm(\alpha+\beta)}')$ constitute a Lie algebra A_2'. But they are not subalgebra of $A(O)$.Our goal is to find the direct sum decomposition of $A(O)$.

Order:

$$\begin{cases} H_1^{\pm} = H_1 \pm \frac{1}{2}H_1', & H_2^{\pm} = H_2 \pm \frac{1}{2}H_2', \\ H_3^{\pm} = H_3 \pm \frac{1}{2}H_3', & \\ E_{\alpha}^{\pm} = E_{\alpha} \pm E_{\alpha}', & E_{-\alpha}^{\pm} = E_{-\alpha} \pm E_{-\alpha}', \\ E_{\beta}^{\pm} = E_{\beta} \pm E_{\beta}', & E_{-\beta}^{\pm} = E_{-\beta} \pm E_{-\beta}'. \\ E_{\alpha+\beta}^{\pm} = E_{\alpha+\beta} \pm E_{\alpha+\beta}' & E_{-(\alpha+\beta)}^{\pm} = E_{-(\alpha+\beta)} \pm E_{-(\alpha+\beta)}' \end{cases} \tag{192}$$

We can get their commutation results shown in Table XIII.

Table XIII means that:

(1) $(H_i^+, E_{\pm\alpha}^+, E_{\pm\beta}^-, E_{\pm(\alpha+\beta)}^+)$ constitute a Lie algebra A_2,

(2)$(H_i^-, E_{\pm\alpha}^-, E_{\pm\beta}^+, E_{\pm(\alpha+\beta)}^-)$ constitute a Lie algebra A_2',and they are subalgebra of $A(O)$. Therefore, order:

$$\begin{array}{ll} H_1 = \frac{1}{8\sqrt{3}}(H_1^+ - H_3^+), & H_1' = \frac{1}{8\sqrt{3}}(H_1^- - H_3^-), \\ H_2 = \frac{1}{32}(2H_2^+ - H_1^+ - H_3^+), & H_2' = \frac{1}{32}(2H_2^+ - H_1^+ - H_3^+), \\ E_{\pm\alpha} = \frac{1}{8\sqrt{6}}E_{\pm\alpha}^+, & E_{\pm\alpha}' = \frac{1}{8\sqrt{6}}E_{\pm\alpha}^-, \\ E_{\pm\beta} = \frac{1}{8\sqrt{6}}E_{\pm\beta}^-, & E_{\pm\beta}' = \frac{1}{8\sqrt{6}}E_{\pm\beta}^+, \\ E_{\pm(\alpha+\beta)} = \frac{1}{8\sqrt{6}}E_{\pm(\alpha+\beta)}^+, & E_{\pm(\alpha+\beta)}' = \frac{1}{8\sqrt{6}}E_{\pm(\alpha+\beta)}^+. \end{array} \tag{193}$$

Then $(H_1, H_2, E_{\pm\alpha}, E_{\pm\beta}, E_{\pm(\alpha+\beta)})$ are the standard basis of A_2;
$(H'_1, H'_2, E'_{\pm\alpha}, E'_{\pm\beta}, E'_{\pm(\alpha+\beta)})$ are the standard basis of A'_2.The relation between these standard basis and the generators of group O are followings.

$$\left\{\begin{aligned}
H_1 =& \frac{1}{8\sqrt{3}}\{T_1 - T_2 \\
&+ \frac{1}{2}[(T_{12} + T_{17} - T_{20} - T_{23}) \\
&- (T_{14} + T_{15} - T_{19} - T_{22})]\}, \\
H_2 =& \frac{1}{32}\{2T_2 - T_1 - T_3 \\
&+ \frac{1}{2}[2(T_{13} + T_{16} - T_{18} - T_{21}) \\
&- (T_{12} + T_{17} - T_{20} - T_{23}) \\
&- (T_{14} + T_{15} - T_{19} - T_{22})]\}, \\
E_\alpha =& + \frac{1}{8\sqrt{6}}[(T_4 + T_5 - T_6 - T_7) \\
&+ (T_{12} - T_{17} - T_{20} + T_{23})], \\
E_{-\alpha} =& + \frac{1}{8\sqrt{6}}[(T_8 + T_9 - T_{10} - T_{11}) \\
&+ (T_{12} - T_{17} + T_{20} - T_{23})], \\
E_\beta =& - \frac{1}{8\sqrt{6}}[(T_4 - T_5 - T_6 + T_7) \\
&- (T_{14} - T_{15} - T_{19} + T_{22})], \\
E_{-\beta} =& - \frac{1}{8\sqrt{6}}[(T_8 - T_9 - T_{10} + T_{11}) \\
&- (T_{14} - T_{15} + T_{19} - T_{22})], \\
E_{+(\alpha+\beta)} =& + \frac{1}{8\sqrt{6}}[(T_4 - T_5 + T_6 - T_7) \\
&+ (T_{13} - T_{16} - T_{18} + T_{21})], \\
E_{-(\alpha+\beta)} =& + \frac{1}{8\sqrt{6}}[(T_8 - T_9 + T_{10} - T_{11}) \\
&+ (T_{13} - T_{16} + T_{18} - T_{21})].
\end{aligned}\right. \tag{194}$$

$$\left\{\begin{aligned}
H'_1 =& \frac{1}{8\sqrt{3}}\{T_1 - T_2 \\
&- \frac{1}{2}[(T_{12} + T_{17} - T_{20} - T_{23}) \\
&- (T_{14} + T_{15} - T_{19} - T_{22})]\}, \\
H'_2 =& \frac{1}{32}\{2T_2 - T_1 - T_3 \\
&- \frac{1}{2}[2(T_{13} + T_{16} - T_{18} - T_{21}) \\
&- (T_{12} + T_{17} - T_{20} - T_{23}) \\
&- (T_{14} + T_{15} - T_{19} - T_{22})]\}, \\
E'_\alpha =& \frac{1}{8\sqrt{6}}[(T_4 + T_5 - T_6 - T_7) \\
&- (T_{12} - T_{17} - T_{20} + T_{23})], \\
E'_{-\alpha} =& \frac{1}{8\sqrt{6}}[(T_8 + T_9 - T_{10} - T_{11}) \\
&- (T_{12} - T_{17} + T_{20} - T_{23})], \\
E'_\beta =& - \frac{1}{8\sqrt{6}}[(T_4 - T_5 - T_6 + T_7) \\
&+ (T_{14} - T_{15} - T_{19} + T_{22})], \\
E'_{-\beta} =& - \frac{1}{8\sqrt{6}}[(T_8 - T_9 - T_{10} + T_{11}) \\
&+ (T_{14} - T_{15} + T_{19} - T_{22})], \\
E'_{+(\alpha+\beta)} =& \frac{1}{8\sqrt{6}}[(T_4 - T_5 + T_6 - T_7) \\
&- (T_{13} - T_{16} - T_{18} + T_{21})], \\
E'_{-(\alpha+\beta)} =& \frac{1}{8\sqrt{6}}[(T_8 - T_9 + T_{10} - T_{11}) \\
&- (T_{13} - T_{16} + T_{18} - T_{21})].
\end{aligned}\right. \tag{195}$$

In addition to the above 2 x 8 = 16 combinations, there are the other eight independent combinations of 24 elements of $A(O)$,

$$\left\{\begin{aligned}
h_1 =& T_{12} + T_{17} + T_{20} + T_{23}, \\
h_2 =& T_{13} + T_{16} + T_{18} + T_{21}, \\
h_3 =& T_{14} + T_{15} + T_{16} + T_{17}, \\
h_4 =& T_4 + T_5 + T_6 + T_7 - T_8 - T_9 - T_{10} - T_{11}.
\end{aligned}\right. \tag{196}$$

$$\left\{\begin{aligned}
X_1 =& E, \\
X_2 =& T_1 + T_2 + T_3, \\
X_3 =& T_4 + T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} + T_{11}, \\
X_4 =& T_{12} + T_{13} + T_{14} + T_{15} + T_{16} + T_{17}, \\
X_5 =& T_{18} + T_{19} + T_{20} + T_{21} + T_{22} + T_{23}.
\end{aligned}\right. \tag{197}$$

Where in the six elements of $(h_1, h_2, h_3, h_4, X4, X_5)$,only 5 are independent. According to the properties of sum of class,these eight combinations are commutation with the elements of A_2 and A'_2.Because

$$\left\{\begin{aligned}
[h_i, X_j] =& 0, \\
[h_i, h_j] \neq& 0, \\
[X_i, X_j] =& 0.
\end{aligned}\right. \tag{198}$$

It means that five X_i is semisimple Lie algebra A_0 and in four h_i (only three are independent) we can constitute some semisimple Lie algbra. We get in directly calculation

$$\begin{cases} [h_1,h_2] = -4h_4, \\ [h_2,h_3] = -4h_4, \\ [h_3,h_1] = -4h_4, \\ [h_3,h_4] = 8h_2 - 8h_1, \\ [h_4,h_1] = 8h_2 - 8h_3, \\ [h_4,h_2] = 8h_3 - 8h_1. \end{cases} \tag{199}$$

Order $Y_1 = h_4, Y_2 = h_1 - h_2, Y_3 = h_2 - h_3; Y' = h_1 + h_2 + h_3 = X_4 + X_5$(separated out),we have

$$\begin{cases} [Y_i, Y'] = 0, (i = 1,2,3), \\ [Y_1,Y_2] = 8Y_2 + 16Y_3, \\ [Y_1,Y_3] = -16Y_2 - 8Y_3, \\ [Y_2,Y_3] = 12Y_1. \end{cases} \tag{200}$$

Order

$$\begin{cases} A = i\frac{\sqrt{3}}{24}Y_1, \\ E_\pm = \frac{\sqrt{3}}{4}[(Y_2 + \frac{1}{2}Y_3) \pm i\frac{\sqrt{3}}{2}Y_3]. \end{cases} \tag{201}$$

We get in directly calculation

$$\begin{cases} [A, E_\pm] = \pm E_\pm, \\ [E_+, E_-] = A. \end{cases} \tag{202}$$

It means that $(A, E_\pm)$ are Lie algebra A_1 and they are standard basis,

$$\begin{cases} A = i\frac{\sqrt{3}}{24}(T_4 + T_5 + T_6 + T_7 - T_8 - T_9 - T_{10} - T_{11}), \\ E_\pm = \frac{\sqrt{3}}{4}[2(T_{12} + T_{17} + T_{20} + T_{23}) \\ \quad - (T_{13} + T_{16} + T_{18} + T_{21}) \\ \quad - (T_{14} + T_{15} + T_{19} + T_{22})] \\ \quad \pm i\frac{3}{4}[(T_{13} + T_{16} + T_{18} + T_{21}) \\ \quad - (T_{14} + T_{15} + T_{19} + T_{22})]. \end{cases} \tag{203}$$

So far we got by direct calculation the direct sum decomposition of A(O) followings.

$$A(O) = \sum_{i=1}^{5} \oplus A_0^i \oplus A_1 \oplus A_2 \oplus A_2'. \tag{204}$$

C. Group algebra $A(O_h)$ of group O_h

According to the definition,$O_h \equiv \{T_i \bigcup T_i\sigma, T_i \in O\}$. Let

$$T_i^\pm = \frac{1}{2}(T_i \pm T_i\sigma) = P_\pm T_i, \tag{205}$$

where $P_\pm = \frac{1}{2}(1 \pm \sigma)$.We have

$$\begin{cases} [T_i^+, T_j^+] = [T_i, T_j]P_+ = C_{ij}^k T_k^+, \\ [T_i^-, T_j^-] = [T_i, T_j]P_- = C_{ij}^k T_k^-, \\ [T_i^+, T_j^-] = 0. \end{cases} \tag{206}$$

Where C_{ij}^k is structure constant of group algebra $A(O)$.Then Formula (206) means that:

(1)$\{T_i^+\}$ and $\{T_i^-\}$ are a group algebra respectively and isomorphic to $A(O)$,denote $A(O^+)$ and $A(O^-)$.

(2) Group algebra $A(O_h)$ are direct sum of $A(O^+)$ and $A(O^-)$,namely $A(O_h) = A(O^+) \oplus A(O^-)$.

D. Group algebra $A(T_d)$ of group T_d

The generators of group T_d are followings.

$$\begin{cases} A(T_d) = \{T_i, \sigma T_\alpha\}; \\ i = 0,1,2,\cdots,11; \\ \alpha = 12,13,\cdots,23; \\ T_i, T_\alpha \in O. \end{cases} \tag{207}$$

We have

$$\begin{cases} [T_i, T_j] = C_{ij}^k T_k, \\ [T_\alpha\sigma, T_i] = C_{\alpha i}^\beta T_\beta\sigma, \\ [T_\alpha\sigma, T_\beta\sigma] = C_{\alpha\beta}^j T_j. \end{cases} \tag{208}$$

Order

$$T_\alpha^d = \sigma T_\alpha, T_i^d = T_i, \tag{209}$$

we have that group algrbra of $T_d = \{T_i^d, T_\alpha^d\}$ is isomorphic to $A(O)$.Their replacement relation is in Formula (209).So we have the direct sum decomposition of $A(T_d)$,

$$A(T_d) = \sum_{i=1}^{5} \oplus A_0^{id} \oplus A_1^d \oplus A_2^d \oplus A_2^{'d}. \tag{210}$$

Put replacement Formula (209) into Formula (194),(195),(197),(203),we have the relation between standard basis of $A(T_d)$ and generators of group T_d shown in Formula (32)-(35).

TABLE XII. Commutation relation between $(H_i, E_{\pm\alpha}, E_{\pm\beta}, E_{\pm(\alpha+\beta)})$ and $(H'_i, E'_{\pm\alpha}, E'_{\pm\beta}, E'_{\pm(\alpha+\beta)})$

$[H_1, E_{\pm\alpha}] = 0$	$[H'_1, E_{\pm\alpha}] = 0$
$[H_1, E_{\pm\beta}] = \pm 2E_{\pm\beta}$	$[H'_1, E_{\pm\beta}] = \mp 4E'_{\pm\beta}$
$[H_1, E_{\pm(\alpha+\beta)}] = \pm 2E_{\pm(\alpha+\beta)}$	$[H'_1, E_{\pm(\alpha+\beta)}] = \pm 4E'_{\pm(\alpha+\beta)}$
$[H_2, E_{\pm\alpha}] = \pm 2E_{\pm\alpha}$	$[H'_2, E_{\pm\alpha}] = \pm 4E'_{\pm\alpha}$
$[H_2, E_{\pm\beta}] = \mp 2E_{\pm\beta}$	$[H'_2, E_{\pm\beta}] = \pm 4E'_{\pm\beta}$
$[H_2, E_{\pm(\alpha+\beta)}] = 0$	$[H'_2, E_{\pm(\alpha+\beta)}] = 0$
$[H_3, E_{\pm\alpha}] = \mp 2E_{\pm\alpha}$	$[H'_3, E_{\pm\alpha}] = \mp 4E'_{\pm\alpha}$
$[H_3, E_{\pm\beta}] = 0$	$[H'_3, E_{\pm\beta}] = 0$
$[H_3, E_{\pm(\alpha+\beta)}] = \mp E_{\pm(\alpha+\beta)}$	$[H'_3, E_{\pm(\alpha+\beta)}] = \mp 4E'_{\pm(\alpha+\beta)}$
$[H_1, E'_{\pm\alpha}] = 0$	$[H'_1, E'_{\pm\alpha}] = 0$
$[H_1, E'_{\pm\beta}] = \pm 2E'_{\pm\beta}$	$[H'_1, E'_{\pm\beta}] = \mp 4E_{\pm\beta}$
$[H_1, E'_{\pm(\alpha+\beta)}] = \pm 2E'_{\pm(\alpha+\beta)}$	$[H'_1, E'_{\pm(\alpha+\beta)}] = \pm 4E_{\pm(\alpha+\beta)}$
$[H_2, E'_{\pm\alpha}] = \pm 2E'_{\pm\alpha}$	$[H'_2, E'_{\pm\alpha}] = \pm 4E_{\pm\alpha}$
$[H_2, E'_{\pm\beta}] = \mp 2E'_{\pm\beta}$	$[H'_2, E'_{\pm\beta}] = \pm 4E_{\pm\beta}$
$[H_2, E'_{\pm(\alpha+\beta)}] = 0$	$[H'_2, E'_{\pm(\alpha+\beta)}] = 0$
$[H_3, E'_{\pm\alpha}] = \mp 2E'_{\pm\alpha}$	$[H'_3, E'_{\pm\alpha}] = \mp 4E_{\pm\alpha}$
$[H_3, E'_{\pm\beta}] = 0$	$[H'_3, E'_{\pm\beta}] = 0$
$[H_3, E'_{\pm(\alpha+\beta)}] = \mp 2E'_{\pm(\alpha+\beta)}$	$[H'_3, E'_{\pm(\alpha+\beta)}] = \mp 4E_{\pm(\alpha+\beta)}$
$[E_\alpha, E_{-\alpha}] = 8H_2 - 8H_3$	$[E'_\alpha, E'_{-\alpha}] = 8H_2 - 8H_3$
$[E_\alpha, E_\beta] = -4E_{\alpha+\beta}$	$[E'_\alpha, E'_\beta] = 4E_{\alpha+\beta}$
$[E_\alpha, E_{-\beta}] = 0$	$[E'_\alpha, E'_{-\beta}] = 0$
$[E_\alpha, E_{(\alpha+\beta)}] = 0$	$[E'_\alpha, E'_{(\alpha+\beta)}] = 0$
$[E_\alpha, E_{-(\alpha+\beta)}] = 4E_{-\beta}$	$[E'_\alpha, E'_{-(\alpha+\beta)}] = 4E_{-\beta}$
$[E_{-\alpha}, E_\beta] = 0$	$[E'_{-\alpha}, E'_\beta] = 0$
$[E_{-\alpha}, E_{-\beta}] = 4E_{-(\alpha+\beta)}$	$[E'_{-\alpha}, E'_{-\beta}] = -4E_{-(\alpha+\beta)}$
$[E_{-\alpha}, E_{(\alpha+\beta)}] = -4E_\beta$	$[E'_{-\alpha}, E'_{(\alpha+\beta)}] = -4E_\beta$
$[E_{-\alpha}, E_{-(\alpha+\beta)}] = 0$	$[E'_{-\alpha}, E'_{-(\alpha+\beta)}] = 0$
$[E_\beta, E_{-\beta}] = 8H_1 - 8H_2$	$[E'_\beta, E'_{-\beta}] = 8H_1 - 8H_2$
$[E_\beta, E_{(\alpha+\beta)}] = 0$	$[E'_\beta, E'_{(\alpha+\beta)}] = 0$
$[E_\beta, E_{-(\alpha+\beta)}] = -4E_{-\alpha}$	$[E'_\beta, E'_{-(\alpha+\beta)}] = 4E_{-\alpha}$
$[E_{-\beta}, E_{(\alpha+\beta)}] = 4E_\alpha$	$[E'_{-\beta}, E'_{(\alpha+\beta)}] = -4E_\alpha$
$[E_{-\beta}, E_{-(\alpha+\beta)}] = 0$	$[E'_{-\beta}, E'_{-(\alpha+\beta)}] = 0$
$[E_{(\alpha+\beta)}, E_{-(\alpha+\beta)}] = 8H_1 - 8H_3$	$[E'_{(\alpha+\beta)}, E'_{-(\alpha+\beta)}] = 8H_1 - 8H_3$
$[E_\alpha, E'_\alpha] = 0$	$[E_{-\alpha}, E'_\alpha] = 4H'_3 - 4H'_2$
$[E_\alpha, E'_{-\alpha}] = 4H'_2 - 4H'_3$	$[E_{-\alpha}, E'_{-\alpha}] = 0$
$[E_\alpha, E'_\beta] = 4E'_{(\alpha+\beta)}$	$[E_{-\alpha}, E'_\beta] = 0$
$[E_\alpha, E'_{-\beta}] = 0$	$[E_{-\alpha}, E'_{-\beta}] = -4E'_{-(\alpha+\beta)}$
$[E_\alpha, E'_{(\alpha+\beta)}] = 0$	$[E_{-\alpha}, E'_{(\alpha+\beta)}] = 4E'_\beta$
$[E_\alpha, E'_{-(\alpha+\beta)}] = -4E'_{-\beta}$	$[E_{-\alpha}, E'_{-(\alpha+\beta)}] = 0$
$[E_\beta, E'_\alpha] = 4E'_{(\alpha+\beta)}$	$[E_{-\beta}, E'_\alpha] = 0$
$[E_\beta, E'_{-\alpha}] = 0$	$[E_{-\beta}, E'_{-\alpha}] = -4E'_{-(\alpha+\beta)}$
$[E_\beta, E'_\beta] = 0$	$[E_{-\beta}, E'_\beta] = 4H'_1 - 4H'_2$
$[E_\beta, E'_{-\beta}] = 4H'_2 - 4H'_1$	$[E_{-\beta}, E'_{-\beta}] = 0$
$[E_\beta, E'_{(\alpha+\beta)}] = 0$	$[E_{-\beta}, E'_{(\alpha+\beta)}] = 4E'_\alpha$
$[E_\beta, E'_{-(\alpha+\beta)}] = 4E'_{-\alpha}$	$[E_{-\beta}, E'_{-(\alpha+\beta)}] = 0$
$[E_{(\alpha+\beta)}, E'_\alpha] = 0$	$[E_{-(\alpha+\beta)}, E'_\alpha] = 4E'_{-\beta}$
$[E_{(\alpha+\beta)}, E'_{-\alpha}] = -4E'_\beta$	$[E_{-(\alpha+\beta)}, E'_{-\alpha}] = 0$
$[E_{(\alpha+\beta)}, E'_\beta] = 0$	$[E_{-(\alpha+\beta)}, E'_\beta] = -4E'_{-\alpha}$
$[E_{(\alpha+\beta)}, E'_{-\beta}] = 4E'_\alpha$	$[E_{-(\alpha+\beta)}, E'_{-\beta}] = 0$
$[E_{(\alpha+\beta)}, E'_{(\alpha+\beta)}] = 0$	$[E_{-(\alpha+\beta)}, E'_{(\alpha+\beta)}] = 4H'_3 - 4H'_1$
$[E_{(\alpha+\beta)}, E'_{-(\alpha+\beta)}] = 4H'_1 - 4H'_3$	$[E_{-(\alpha+\beta)}, E'_{-(\alpha+\beta)}] = 0$

TABLE XIII. Commutation relation between $(H_i^+, E^+_{\pm\alpha}, E^-_{\pm\beta}, E^+_{\pm(\alpha+\beta)})$ and $(H_i^-, E^-_{\pm\alpha}, E^+_{\pm\beta}, E^-_{\pm(\alpha+\beta)})$

$[H_1^+, E^+_{\pm\alpha}] = 0$	$[H_1^-, E^-_{\pm\alpha}] = 0$
$[H_1^+, E^-_{\pm\beta}] = \pm 4E^-_{\pm\beta}$	$[H_1^-, E^+_{\pm\beta}] = \pm 4E^+_{\pm\beta}$
$[H_1^+, E^+_{\pm(\alpha+\beta)}] = \pm 4E^+_{\pm(\alpha+\beta)}$	$[H_1^-, E^-_{\pm(\alpha+\beta)}] = \pm 4E^+_{\pm(\alpha+\beta)}$
$[H_2^+, E^+_{\pm\alpha}] = \pm 4E^+_{\pm\alpha}$	$[H_2^-, E^-_{\pm\alpha}] = \pm 4E^-_{\pm\alpha}$
$[H_2^+, E^-_{\pm\beta}] = \mp 4E^-_{\pm\beta}$	$[H_2^-, E^+_{\pm\beta}] = \mp 4E^+_{\pm\beta}$
$[H_2^+, E^+_{\pm(\alpha+\beta)}] = 0$	$[H_2^-, E^-_{\pm(\alpha+\beta)}] = 0$
$[H_3^+, E^+_{\pm\alpha}] = \mp 4E^+_{\pm\alpha}$	$[H_3^-, E^-_{\pm\alpha}] = \mp 4E^-_{\pm\alpha}$
$[H_3^+, E^+_{\pm\beta}] = 0$	$[H_3^-, E^-_{\pm\beta}] = 0$
$[H_3^+, E^+_{\pm(\alpha+\beta)}] = \mp 4E^+_{\pm(\alpha+\beta)}$	$[H_3^-, E^-_{\pm(\alpha+\beta)}] = \mp 4E^-_{\pm(\alpha+\beta)}$
$[H_1^+, E^-_{\pm\alpha}] = 0$	$[H_1^-, E^+_{\pm\alpha}] = 0$
$[H_1^+, E^+_{\pm\beta}] = 0$	$[H_1^-, E^-_{\pm\beta}] = 0$
$[H_1^+, E^-_{\pm(\alpha+\beta)}] = 0$	$[H_1^-, E^+_{\pm(\alpha+\beta)}] = 0$
$[H_2^+, E^-_{\pm\alpha}] = 0$	$[H_2^-, E^+_{\pm\alpha}] = 0$
$[H_2^+, E^+_{\pm\beta}] = 0$	$[H_2^-, E^-_{\pm\beta}] = 0$
$[H_2^+, E^-_{\pm(\alpha+\beta)}] = 0$	$[H_2^-, E^+_{\pm(\alpha+\beta)}] = 0$
$[H_3^+, E^-_{\pm\alpha}] = 0$	$[H_3^-, E^+_{\pm\alpha}] = 0$
$[H_3^+, E^+_{\pm\beta}] = 0$	$[H_3^-, E^-_{\pm\beta}] = 0$
$[H_3^+, E^-_{\pm(\alpha+\beta)}] = 0$	$[H_3^-, E^+_{\pm(\alpha+\beta)}] = 0$
$[E^+_\alpha, E^+_{-\alpha}] = 16H_2^+ - 16H_3^+$	$[E^-_\alpha, E^-_{-\alpha}] = 16H_2^- - 16H_3^-$
$[E^+_\alpha, E^-_\beta] = -8E^+_{\alpha+\beta}$	$[E^-_\alpha, E^+_\beta] = -8E^-_{\alpha+\beta}$
$[E^+_\alpha, E^-_{-\beta}] = 0$	$[E^-_\alpha, E^+_{-\beta}] = 0$
$[E^+_\alpha, E^+_{(\alpha+\beta)}] = 0$	$[E^-_\alpha, E^-_{(\alpha+\beta)}] = 0$
$[E^+_\alpha, E^+_{-(\alpha+\beta)}] = 8E^-_{-\beta}$	$[E^-_\alpha, E^-_{-(\alpha+\beta)}] = 8E^+_{-\beta}$
$[E^+_{-\alpha}, E^-_\beta] = 0$	$[E^-_{-\alpha}, E^+_\beta] = 0$
$[E^+_{-\alpha}, E^-_{-\beta}] = 8E^+_{-(\alpha+\beta)}$	$[E^-_{-\alpha}, E^+_{-\beta}] = 8E^-_{-(\alpha+\beta)}$
$[E^+_{-\alpha}, E^+_{(\alpha+\beta)}] = -8E^-_\beta$	$[E^-_{-\alpha}, E^-_{(\alpha+\beta)}] = -8E_\beta$
$[E^+_{-\alpha}, E^+_{-(\alpha+\beta)}] = 0$	$[E^-_{-\alpha}, E^-_{-(\alpha+\beta)}] = 0$
$[E^-_\beta, E^-_{-\beta}] = 16H_1^+ - 16H_2^+$	$[E^+_\beta, E^+_{-\beta}] = 16H_1^- - 16H_2^-$
$[E^-_\beta, E^+_{(\alpha+\beta)}] = 0$	$[E^+_\beta, E^-_{(\alpha+\beta)}] = 0$
$[E^-_\beta, E^+_{-(\alpha+\beta)}] = -8E^+_{-\alpha}$	$[E^+_\beta, E^-_{-(\alpha+\beta)}] = -8E^-_{-\alpha}$
$[E^-_{-\beta}, E^+_{(\alpha+\beta)}] = 8E^+_\alpha$	$[E^+_{-\beta}, E^-_{(\alpha+\beta)}] = 8E^-_\alpha$
$[E^-_{-\beta}, E^+_{-(\alpha+\beta)}] = 0$	$[E^+_{-\beta}, E'_{-(\alpha+\beta)}] = 0$
$[E^+_{(\alpha+\beta)}, E^+_{-(\alpha+\beta)}] = 16H_1^+ - 16H_3^+$	$[E^-_{(\alpha+\beta)}, E^-_{-(\alpha+\beta)}] = 16H_1^- - 16H_3^-$
$[E^+_\alpha, E^-_\alpha] = 0$	$[E^+_{-\alpha}, E^-_\alpha] = 0$
$[E^+_\alpha, E^-_{-\alpha}] = 0$	$[E^+_{-\alpha}, E^-_{-\alpha}] = 0$
$[E^+_\alpha, E^+_\beta] = 0$	$[E^+_{-\alpha}, E^+_\beta] = 0$
$[E^+_\alpha, E^+_{-\beta}] = 0$	$[E^+_{-\alpha}, E^+_{-\beta}] = 0$
$[E^+_\alpha, E^-_{(\alpha+\beta)}] = 0$	$[E^+_{-\alpha}, E^-_{(\alpha+\beta)}] = 0$
$[E^+_\alpha, E^-_{-(\alpha+\beta)}] = 0$	$[E^+_{-\alpha}, E^-_{-(\alpha+\beta)}] = 0$
$[E^-_\beta, E^-_\alpha] = 0$	$[E^-_{-\beta}, E^-_\alpha] = 0$
$[E^-_\beta, E^-_{-\alpha}] = 0$	$[E^-_{-\beta}, E^-_{-\alpha}] = 0$
$[E^-_\beta, E^+_\beta] = 0$	$[E^-_{-\beta}, E^+_\beta] = 0$
$[E^-_\beta, E^+_{-\beta}] = 0$	$[E^-_{-\beta}, E^+_{-\beta}] = 0$
$[E^-_\beta, E^-_{(\alpha+\beta)}] = 0$	$[E^-_{-\beta}, E^-_{(\alpha+\beta)}] = 0$
$[E^-_\beta, E^-_{-(\alpha+\beta)}] = 0$	$[E^-_{-\beta}, E^-_{-(\alpha+\beta)}] = 0$
$[E^+_{(\alpha+\beta)}, E^-_\alpha] = 0$	$[E^+_{-(\alpha+\beta)}, E^-_\alpha] = 0$
$[E^+_{(\alpha+\beta)}, E^-_{-\alpha}] = 0$	$[E^+_{-(\alpha+\beta)}, E^-_{-\alpha}] = 0$
$[E^+_{(\alpha+\beta)}, E^+_\beta] = 0$	$[E^+_{-(\alpha+\beta)}, E^+_\beta] = 0$
$[E^+_{(\alpha+\beta)}, E^+_{-\beta}] = 0$	$[E^+_{-(\alpha+\beta)}, E^+_{-\beta}] = 0$
$[E^+_{(\alpha+\beta)}, E^-_{(\alpha+\beta)}] = 0$	$[E^+_{-(\alpha+\beta)}, E^-_{(\alpha+\beta)}] = 0$
$[E^+_{(\alpha+\beta)}, E^-_{-(\alpha+\beta)}] = 0$	$[E^+_{-(\alpha+\beta)}, E^-_{-(\alpha+\beta)}] = 0$

Table 14 The multiplication table of O,S_4,and Td

	E	T_{1}	T_{2}	T_{3}	T_{4}	T_{5}	T_{6}	T_{7}	T_{8}	T_{9}	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}	T_{17}	T_{18}	T_{19}	T_{20}	T_{21}	T_{22}	T_{23}
E	E	T_{1}	T_{2}	T_{3}	T_{4}	T_{5}	T_{6}	T_{7}	T_{8}	T_{9}	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}	T_{17}	T_{18}	T_{19}	T_{20}	T_{21}	T_{22}	T_{23}
T_{1}	T_{1}	E	T_{3}	T_{2}	T_{7}	T_{6}	T_{5}	T_{4}	T_{10}	T_{11}	T_{8}	T_{9}	T_{17}	T_{21}	T_{22}	T_{19}	T_{18}	T_{12}	T_{16}	T_{15}	T_{23}	T_{13}	T_{14}	T_{20}
T_{2}	T_{2}	T_{3}	E	T_{1}	T_{5}	T_{4}	T_{7}	T_{6}	T_{11}	T_{10}	T_{9}	T_{8}	T_{23}	T_{16}	T_{19}	T_{22}	T_{13}	T_{20}	T_{21}	T_{14}	T_{17}	T_{18}	T_{15}	T_{12}
T_{3}	T_{3}	T_{2}	T_{1}	E	T_{6}	T_{7}	T_{4}	T_{5}	T_{9}	T_{8}	T_{11}	T_{10}	T_{20}	T_{18}	T_{15}	T_{14}	T_{21}	T_{23}	T_{13}	T_{22}	T_{12}	T_{16}	T_{19}	T_{17}
T_{4}	T_{4}	T_{6}	T_{7}	T_{5}	T_{8}	T_{10}	T_{11}	T_{9}	E	T_{2}	T_{3}	T_{1}	T_{13}	T_{15}	T_{23}	T_{12}	T_{19}	T_{18}	T_{22}	T_{20}	T_{16}	T_{14}	T_{17}	T_{21}
T_{5}	T_{5}	T_{7}	T_{6}	T_{4}	T_{11}	T_{9}	T_{8}	T_{10}	T_{2}	E	T_{1}	T_{3}	T_{16}	T_{22}	T_{12}	T_{23}	T_{14}	T_{21}	T_{15}	T_{17}	T_{13}	T_{19}	T_{20}	T_{18}
T_{6}	T_{6}	T_{4}	T_{5}	T_{7}	T_{9}	T_{11}	T_{10}	T_{8}	T_{3}	T_{1}	E	T_{2}	T_{18}	T_{14}	T_{17}	T_{20}	T_{22}	T_{13}	T_{19}	T_{12}	T_{21}	T_{15}	T_{23}	T_{16}
T_{7}	T_{7}	T_{5}	T_{4}	T_{6}	T_{10}	T_{8}	T_{9}	T_{11}	T_{1}	T_{3}	T_{2}	E	T_{21}	T_{19}	T_{20}	T_{17}	T_{15}	T_{16}	T_{14}	T_{23}	T_{18}	T_{22}	T_{12}	T_{13}
T_{8}	T_{8}	T_{11}	T_{9}	T_{10}	E	T_{3}	T_{1}	T_{2}	T_{4}	T_{7}	T_{5}	T_{6}	T_{15}	T_{12}	T_{21}	T_{13}	T_{20}	T_{22}	T_{17}	T_{16}	T_{19}	T_{23}	T_{18}	T_{14}
T_{9}	T_{9}	T_{10}	T_{8}	T_{11}	T_{3}	E	T_{2}	T_{1}	T_{6}	T_{5}	T_{7}	T_{4}	T_{14}	T_{20}	T_{16}	T_{18}	T_{12}	T_{19}	T_{23}	T_{21}	T_{22}	T_{17}	T_{13}	T_{15}
T_{10}	T_{10}	T_{9}	T_{11}	T_{8}	T_{1}	T_{2}	E	T_{3}	T_{7}	T_{4}	T_{6}	T_{5}	T_{19}	T_{17}	T_{13}	T_{21}	T_{23}	T_{14}	T_{12}	T_{18}	T_{15}	T_{20}	T_{16}	T_{22}
T_{11}	T_{11}	T_{8}	T_{10}	T_{9}	T_{2}	T_{1}	T_{3}	E	T_{5}	T_{6}	T_{4}	T_{7}	T_{22}	T_{23}	T_{18}	T_{16}	T_{17}	T_{15}	T_{20}	T_{13}	T_{14}	T_{12}	T_{21}	T_{19}
T_{12}	T_{12}	T_{17}	T_{20}	T_{23}	T_{15}	T_{14}	T_{22}	T_{19}	T_{13}	T_{16}	T_{21}	T_{18}	E	T_{8}	T_{5}	T_{4}	T_{9}	T_{1}	T_{11}	T_{7}	T_{2}	T_{10}	T_{6}	T_{3}
T_{13}	T_{13}	T_{18}	T_{16}	T_{21}	T_{12}	T_{23}	T_{17}	T_{20}	T_{15}	T_{19}	T_{14}	T_{22}	T_{4}	E	T_{10}	T_{8}	T_{2}	T_{6}	T_{1}	T_{9}	T_{7}	T_{3}	T_{11}	T_{5}
T_{14}	T_{14}	T_{19}	T_{22}	T_{15}	T_{18}	T_{16}	T_{13}	T_{21}	T_{20}	T_{12}	T_{17}	T_{23}	T_{9}	T_{6}	E	T_{3}	T_{5}	T_{10}	T_{4}	T_{1}	T_{8}	T_{7}	T_{2}	T_{11}
T_{15}	T_{15}	T_{22}	T_{19}	T_{14}	T_{13}	T_{21}	T_{18}	T_{16}	T_{12}	T_{20}	T_{23}	T_{17}	T_{8}	T_{4}	T_{3}	E	T_{7}	T_{11}	T_{6}	T_{2}	T_{9}	T_{5}	T_{1}	T_{10}
T_{16}	T_{16}	T_{21}	T_{13}	T_{18}	T_{23}	T_{12}	T_{20}	T_{17}	T_{22}	T_{14}	T_{19}	T_{15}	T_{5}	T_{2}	T_{9}	T_{11}	E	T_{7}	T_{3}	T_{10}	T_{6}	T_{1}	T_{8}	T_{4}
T_{17}	T_{17}	T_{12}	T_{23}	T_{20}	T_{19}	T_{22}	T_{14}	T_{15}	T_{21}	T_{18}	T_{13}	T_{16}	T_{1}	T_{10}	T_{6}	T_{7}	T_{11}	E	T_{9}	T_{4}	T_{3}	T_{8}	T_{5}	T_{2}
T_{18}	T_{18}	T_{13}	T_{21}	T_{16}	T_{20}	T_{17}	T_{23}	T_{12}	T_{14}	T_{22}	T_{15}	T_{19}	T_{6}	T_{3}	T_{11}	T_{9}	T_{1}	T_{4}	T_{2}	T_{8}	T_{5}	E	T_{10}	T_{7}
T_{19}	T_{19}	T_{14}	T_{15}	T_{22}	T_{21}	T_{13}	T_{16}	T_{18}	T_{17}	T_{23}	T_{20}	T_{12}	T_{10}	T_{7}	T_{2}	T_{1}	T_{4}	T_{9}	T_{5}	T_{3}	T_{11}	T_{6}	E	T_{8}
T_{20}	T_{20}	T_{23}	T_{12}	T_{17}	T_{14}	T_{15}	T_{19}	T_{22}	T_{18}	T_{21}	T_{16}	T_{13}	T_{3}	T_{9}	T_{7}	T_{6}	T_{8}	T_{2}	T_{10}	T_{5}	T_{1}	T_{11}	T_{4}	E
T_{21}	T_{21}	T_{16}	T_{18}	T_{13}	T_{17}	T_{20}	T_{12}	T_{23}	T_{19}	T_{15}	T_{22}	T_{14}	T_{7}	T_{1}	T_{8}	T_{10}	T_{3}	T_{5}	E	T_{11}	T_{4}	T_{2}	T_{9}	T_{6}
T_{22}	T_{22}	T_{15}	T_{14}	T_{19}	T_{16}	T_{18}	T_{21}	T_{13}	T_{23}	T_{17}	T_{12}	T_{20}	T_{11}	T_{5}	T_{1}	T_{2}	T_{6}	T_{8}	T_{7}	E	T_{10}	T_{4}	T_{3}	T_{9}
T_{23}	T_{23}	T_{20}	T_{17}	T_{12}	T_{22}	T_{19}	T_{15}	T_{14}	T_{16}	T_{13}	T_{18}	T_{21}	T_{2}	T_{11}	T_{4}	T_{5}	T_{10}	T_{3}	T_{8}	T_{6}	E	T_{9}	T_{7}	T_{1}

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