The prime addition table contains all even numbers greater than 2

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Abstract

In this paper, I prove that the prime addition table contains all even numbers greater than 2.

MSC classes: Primary 11A41, Secondary 11B13,58D19

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Introduction

A prime addition table is a set of all two prime sums.Because we have only known finite primes now,it seems that there is only incomplete set of all two prime sums. However,we know that there are infinite primes(Euclid), we also know that the distribution of primes is asymptotic and follows the Prime Number Theorem(C.Gauss and A.M.Legendre in 1792;P. Chebyshev in 1849;B.Rosser in 1941;A. Selberg and P. Erdos in 1949;). Then, we can get some properties of the set of two prime sums, though not all of them. My research shows that these properties are sufficient to prove that the set of all two prime sums contains all even numbers greater than 2.

In this paper I study it in three steps.First step (Section 1), we got a strictly ordered sequence of prime numbers,$P = \{p_i; i = 1, 2, 3, ...\}$ .I studied the meaning of subscript i,which is the i^{th} layer of nature numbers.I introduce the concept of odd holes to study the internal structure of prime sequence.Second step (Section 2), we construct a set of all two prime sums by using the arithmetic addition table of prime sequence. However, this prime addition table can not intuitively show the situation containing even numbers. I constructed an equivalent table of prime additions,and the situation containing even numbers is clearly presented in the table. There are many even number holes not included in the equivalent prime addition table. It is easy to see that if an even number is not included in the addition table (that is, Goldbach's exceptions), it is equivalent to the existence of through holes in the equivalent prime addition table. The condition of forming a through hole is studied, and it is found that it is a translational symmetry of prime sequence. Third step (Section 3), we study the translation symmetry of sequence. The condition for the existence of Goldbach's exceptions is that the prime sequence and its corresponding non-prime odd sequence have the complementary symmetry of translating these exceptions. I prove that the PNT negates the existence of this symmetry. This proves that the prime addition table contains all even numbers greater than 2.

1 The Generation of Primes and the Prime Numbers Theorem

The set of natural numbers is $N = \{i; i = 1, 2, 3, 4, ...\}$.

Definition 1.0.1 *For any nature number $m \in N$,the* m*'s coset of N,$S_m \equiv \{mi; i = 1, 2, 3, 4, ...\}$.*

Therefore,the coset $S_1 = N$,which is a set of natural numbers. For $m = 2$,the coset $S_2 = \{2, 4, 6, ...\}$,that is the all even numbers. For $m = 3$,the coset $S_3 = \{3, 6, 9, ...\}$,it is the set obtained by multiplying each natural number by *3*. The set of all other natural numbers and so on. If a number m can be decomposed into the product of l and k, $m = l \times k$, then $S_m = \{ilk; i = 1, 2, 3, 4, ...\}$, so $S_m \subset S_k$ and $S_m \subset S_l$,are subsets of S_l and S_k. For example, $S_6 = \{6, 12, 18, ...\}$, because $6 = 2 \times 3$, $S_6 \subset S_2$ and $S_6 \subset S_3$. Obviously, only the cosets of primes are the true subsets of natural numbers.

1.1 The Generation of Primes

Table 1 The table of prime numbers coset

N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
S2		2		4		6		8		10		12		14		16		18	
S3			3			6			9			12			15			18	
S5					5					10					15				
S7							7							14					
S11											11								
S13													13						
S17																	17		
S19																			19
S23																			
S29																			
S31																			
S37																			
...																			

N	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	...
S2	20		22		24		26		28		30		32		34		36		
S3		21			24			27			30			33			36		
S5	20					25					30					35			
S7		21							28							35			
S11			22											33					
S13							26												
S17															34				
S19																			
S23				23															
S29										29									
S31												31							
S37																		37	
...																			

We can use the cosets of primes to form the natural number N. We have the first coset S_2. From $2 \to 4$, the number 3 is missing in the middle, and it is impossible to find the number 3 in S_2 , so we need to add another coset of number 3 ,S_3. We have S_2 and S_3 two cosets now. In $S_2 \cup S_3$, from $2 \to 3 \to 4 \to 6$, the number 5 is missing, we need to add another coset of number 5, S_5. In $S_2 \cup S_3 \cup S_5$, from $2 \to 3 \to 4 \to 5 \to 6 \to 8$,the number 7 is missing, we need to add another coset of number 7, S_7. In $S_2 \cup S_3 \cup S_5 \cup S_7$,from $2 \to 3 \to 4 \to 5 \to 6 \to 7 \to 8 \to 9 \to 10 \to 12$, the number 11 is missing, we need to add another coset of number 11, S_{11}. Generally, when we have found the natural number $2 \to 3 \to 4 \to 5 \to 6 \to .... \to p_n$ in the $\bigcup\limits_{i=1}^{n} S_{p_i}$,and then to find the following natural number $p_n+1 \to p_n+2 \to p_n+3 \to ... \to p_n+m$, until p_n+m is not included.It means that such p_n+m are not expressed as the products of another numbers.Therefore,this p_n+m is the $(n+1)^{th}$ prime, denoted by $p_{n+1} = p_n + m$.We need to add a prime coset $S_{p_{n+1}}$.

Primes are generated sequentially. Each prime coset is in a row. The s-mallest prime $p_1 = 2$ is in the first row. The row number n has a one-to-one correspondence with the prime, that is, $p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7, ...$ Line *n+1* must be the next row of line n, and all the primes form a strictly ordered set,

$$P = \{p_i; i = 1, 2, 3, 4, ...\} = \{2, 3, 5, 7, 11, 13, 17, ...\}. \tag{1}$$

The union of finite n row prime cosets is $\bigcup\limits_{i=1}^{n} S_{p_i}$.No matter how big n is, if the all next natural numbers can be found in the union of the coset of finite n rows prime, then this n is too special, which is unreasonable. Therefore,$N_n \equiv \{1, \bigcup\limits_{i=1}^{n} S_{p_i}\} \subset N$,it can not be constructed as a natural number set. Only n to infinity can fill natural numbers, that is, primes have infinite rows,

$$N = \lim_{n\to\infty} N_n = \{1, \bigcup_{i=1}^{\infty} S_{p_i}\}. \tag{2}$$

1.2 The Element Density and the Prime Number Theorem

We study the density of elements in the prime coset table(Table 1) to show the importance of the coset table. The probability of each row element appearing in Table 1 is same. For example, S_2, the density of elements appearing in N is $\frac{1}{2}$, and the density of S_3 is $\frac{1}{3}$.and the density of S_5 is $\frac{1}{5}$. In general, the density of S_{p_n} is $\frac{1}{p_n}$. In the prime coset table(Table 1), the density decreases from top to bottom by the reciprocal of primes. The number of elements in each column is non-uniform, and the density before column n is also non-uniform. In Table 1, the primes are the starting elements, and the columns in which the primes are located have only one element, so the primes are truncated. We can denote the number before p_n (including p_n) as N_{p_n}.It is easy to get

$$N_{p_n} = \sum_{i=1}^{n} [\frac{p_n}{p_i}]. \tag{3}$$

The total seat before p_n (though many of the positions are empty) is $V_{p_n} = p_n n$. The density of the element before p_n is ρ_n,so

$$\rho_n \equiv \frac{N_{p_n}}{V_{p_n}} = \frac{1}{p_n n} \sum_{i=1}^{n} [\frac{p_n}{p_i}]. \tag{4}$$

When n is larger,

$$\rho_n \approx \frac{1}{n}\sum_{i=1}^{n}\frac{1}{p_i}. \tag{5}$$

Given the primes P_n, the density for every p_n prime can be calculated by the Formula (5). Draw the density $\rho \sim n$ curve of 1229 primes within the first 10000, as shown in Figure 1.

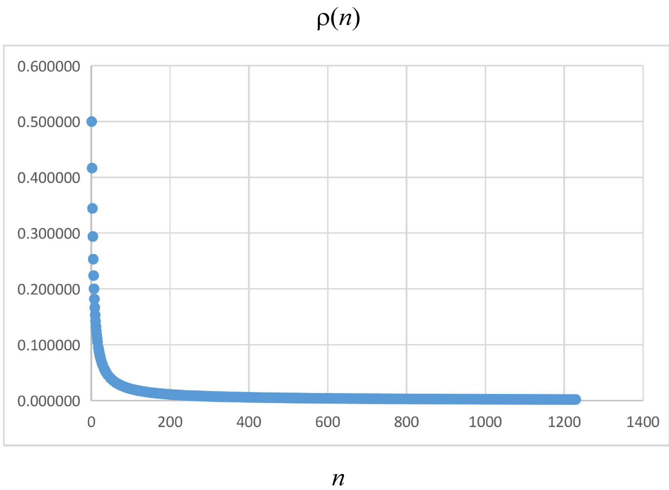

Figure 1: The density of number in the coset table of primes

This density ρ_n is obviously related to n and it is smoothly. With the following recursive relationship,

$$\rho_n = \frac{n-1}{n}\rho_{n-1} + \frac{1}{n^2(\ln n + \ln\ln n - 1)}, \tag{6}$$

The density curve of Figure 1 is well fitted, and the error is about $-3.9 \times 10^{-8}\%$ when $n = 1229$.The formula (5) can be wrote by

$$\rho_n \approx \frac{n-1}{n}\rho_{n-1} + \frac{1}{np_n}. \tag{7}$$

Comparing Formula (7) with (6),we got

$$p_n \approx n(\ln n + \ln\ln n - 1) \approx n\ln n. \tag{8}$$

This result accords with the Prime Number Theorem[1].Acoording to the PNT(8),we got following two corollaries.

Corollary 1.2.1 *The distribution of prime numbers is non-uniform and the density decreases gradually.*

Proof 1 *There are n primes within p_n,and the primes density $\pi(p_n)$ is*

$$\pi(p_n) = \frac{n}{p_n}. \tag{9}$$

According to the PNT(8),

$$\pi(p_n) \sim \frac{1}{\ln n + \ln\ln n - 1} \sim \frac{1}{\ln n}, \tag{10}$$

which means that the density is approximately function of n and the distribution of primes is not uniform. With the increase of n, the density of primes decreases gradually with $\frac{1}{\ln n}$.

Corollary 1.2.2 *For large n,the distribution function of primes is approximately smooth without mutation intervals.*

Proof 2 *For large $n \gg 1$,the PNT(8) is approximately continuous,and we can get the derivative of the prime formula (8)*

$$\frac{dp_n}{dn} \approx \ln n + \ln\ln n + \frac{1}{\ln n}; n \geq 2. \tag{11}$$

The Figure 2 is the curve of Formula (11).

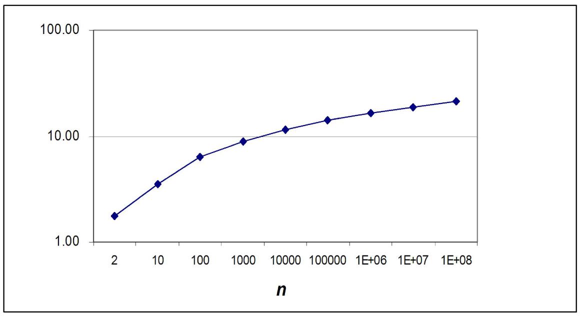

Figure 2: The Derivative of Prime

We can see that the derivative is greater than zero and increases gradually. Therefore, there is no inflection point and the size of primes increases monotonously with n.It means that the distribution function of primes is approximately smooth without mutation intervals for large n.

1.3 The Structure of Prime Sequence

Therefore,we have obtained a strictly ordered and infinitely long primes sequence.In order to study the properties of the primes sequence,we introduced finitely length primes sequence.

Definition 1.3.1 $P_m = \{p_i; i = 1, 2, 3, ..., m\}$.

It is a subset of the primes sequence P.Obviously,there are many non-prime odd numbers between primes.

Definition 1.3.2 *The odd holes of P_m are the non-prime odds between p_{i-1} and p_i,where $i \le m$.The deep of an odd hole is denoted by k_i,it is defined below,*

$$k_i \equiv \frac{1}{2}(p_i - p_{i-1}) - 1. \tag{12}$$

It is number of non-prime odds between p_{i-1} and p_i.Therefore,the $k_i = 0$ for twin primes.In $[p_{i-1}, p_i]$,the non-prime odd are

$$c_{ij} = p_{i-1} + 2j; j = 1, 2, 3, ..., k_i, \tag{13}$$

$$C_i = \{c_{ij}; j = 1, 2, 3, ..., k_i\}. \tag{14}$$

There is not c_{ij} when $k_i = 0$.And total deep of P_m is denoted by K_m,we have

$$K_m \equiv \sum_{i=2}^{m} k_i = \frac{1}{2}(p_m - 3) - (m - 1). \tag{15}$$

For example,$m = 1229$,$p_{1229} = 9973$,$K_{1229} = 3757$,there are 3757 non-prime odds from $p_1 = 2$ to $p_{1229} = 9973$.For large m,according to the PNT(8),

$$K_m \approx \frac{1}{2} m \ln m \gg m. \tag{16}$$

That is said that the deep of odd holes is larger than the numbers of primes in P_m for larger m.

In $[3, p_m]$,there are K_m non-prime odds and $(m - 1)$ primes.Adding odd number 1,the $K_m + 1$ non-prime odds sequence less than p_m is

$$\begin{aligned} O_{P_m} &= \{1\} + \{C_i; i = 1, 2, 3, ..., m\}, && (17)\\ &= \{1, c_{ij}; j = 1, 2, 3, ..., k_i; i = 1, 2, 3, ..., m\}, && (18)\\ &= \{o_l; l = 0, 1, 2, 3, ..., K_m\}, && (19)\\ &= \{1, 9, 15, ..., o_{K_m-1}, o_{K_m}\}. && (20) \end{aligned}$$

Where $o_0 = 1, o_{l-1} < o_l$,and o_l is the odd number of all c_{ij} arranged from small to large.

2 The Prime Addition Table

In order to obtain the every sum of all two primes, the all prime sequence are arranged in the first row and the first column of the table, and the prime addition table 2 is made according to the arithmetic addition rule.

Table 2 The table of primes addition

i	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	...
pi	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	
2	4	5	7	9	13	15	19	21	25	31	33	39	43	45	49	55	61	63	69	73	75	81	
3	5	6	8	10	14	16	20	22	26	32	34	40	44	46	50	56	62	64	70	74	76	82	
5	7	8	10	12	16	18	22	24	28	34	36	42	46	48	52	58	64	66	72	76	78	84	
7	9	10	12	14	18	20	24	26	30	36	38	44	48	50	54	60	66	68	74	78	80	86	
11	13	14	16	18	22	24	28	30	34	40	42	48	52	54	58	64	70	72	78	82	84	90	
13	15	16	18	20	24	26	30	32	36	42	44	50	54	56	60	66	72	74	80	84	86	92	
17	19	20	22	24	28	30	34	36	40	46	48	54	58	60	64	70	76	78	84	88	90	96	
19	21	22	24	26	30	32	36	38	42	48	50	56	60	62	66	72	78	80	86	90	92	98	
23	25	26	28	30	34	36	40	42	46	52	54	60	64	66	70	76	82	84	90	94	96	102	
29	31	32	34	36	40	42	46	48	52	58	60	66	70	72	76	82	88	90	96	100	102	108	
31	33	34	36	38	42	44	48	50	54	60	62	68	72	74	78	84	90	92	98	102	104	110	
37	39	40	42	44	48	50	54	56	60	66	68	74	78	80	84	90	96	98	104	108	110	116	
41	43	44	46	48	52	54	58	60	64	70	72	78	82	84	88	94	100	102	108	112	114	120	
43	45	46	48	50	54	56	60	62	66	72	74	80	84	86	90	96	102	104	110	114	116	122	
47	49	50	52	54	58	60	64	66	70	76	78	84	88	90	94	100	106	108	114	118	120	126	
53	55	56	58	60	64	66	70	72	76	82	84	90	94	96	100	106	112	114	120	124	126	132	
59	61	62	64	66	70	72	76	78	82	88	90	96	100	102	106	112	118	120	126	130	132	138	
61	63	64	66	68	72	74	78	80	84	90	92	98	102	104	108	114	120	122	128	132	134	140	
67	69	70	72	74	78	80	84	86	90	96	98	104	108	110	114	120	126	128	134	138	140	146	
71	73	74	76	78	82	84	88	90	94	100	102	108	112	114	118	124	130	132	138	142	144	150	
73	75	76	78	80	84	86	90	92	96	102	104	110	114	116	120	126	132	134	140	144	146	152	
79	81	82	84	86	90	92	96	98	102	108	110	116	120	122	126	132	138	140	146	150	152	158	
...																							

The elements in Table 2 are the sum of the corresponding rows and column primes p_i and p_j, denoted by

$$h_{ij} = p_i + p_j; i, j = 1, 2, 3, 4, ... \tag{21}$$

It is easy to get

$$h_{ij} = h_{ji}. \tag{22}$$

It is the symmetry of rows and columns.

Definition 2.0.3 *The set of the evens in the primes addition table denoted by*

$$H \equiv \{h_{11}, h_{ij}; i, j = 2, 3, 4, ...\} = \{4, h_{ij}; i, j = 2, 3, 4, ...\}. \tag{23}$$

Theorem 2.0.4 *H contains the sums of all two primes.*

Proof 3 *Because the even element $h_{ij} = p_i + p_j$,p_i and p_j takes all primes, the sums of all possible pairwise primes is contained in H.*

The Theorem 2.0.1 seems to be simple,but it is very important.It tell us that the primes addition table constructs a evens set H which includes all possible sum of two primes.Only need to prove $S_2 \subset H$,that is all evens in H,we can prove the Goldbach's conjecture.

2.1 The Equivalent Prime Additive Table

Definition 2.1.1 *Hole.Given a set S, there is an even E, which satisfies $E \in S_2$,but $E \notin S$ and then the E is called even hole of S, which is referred to as hole.*

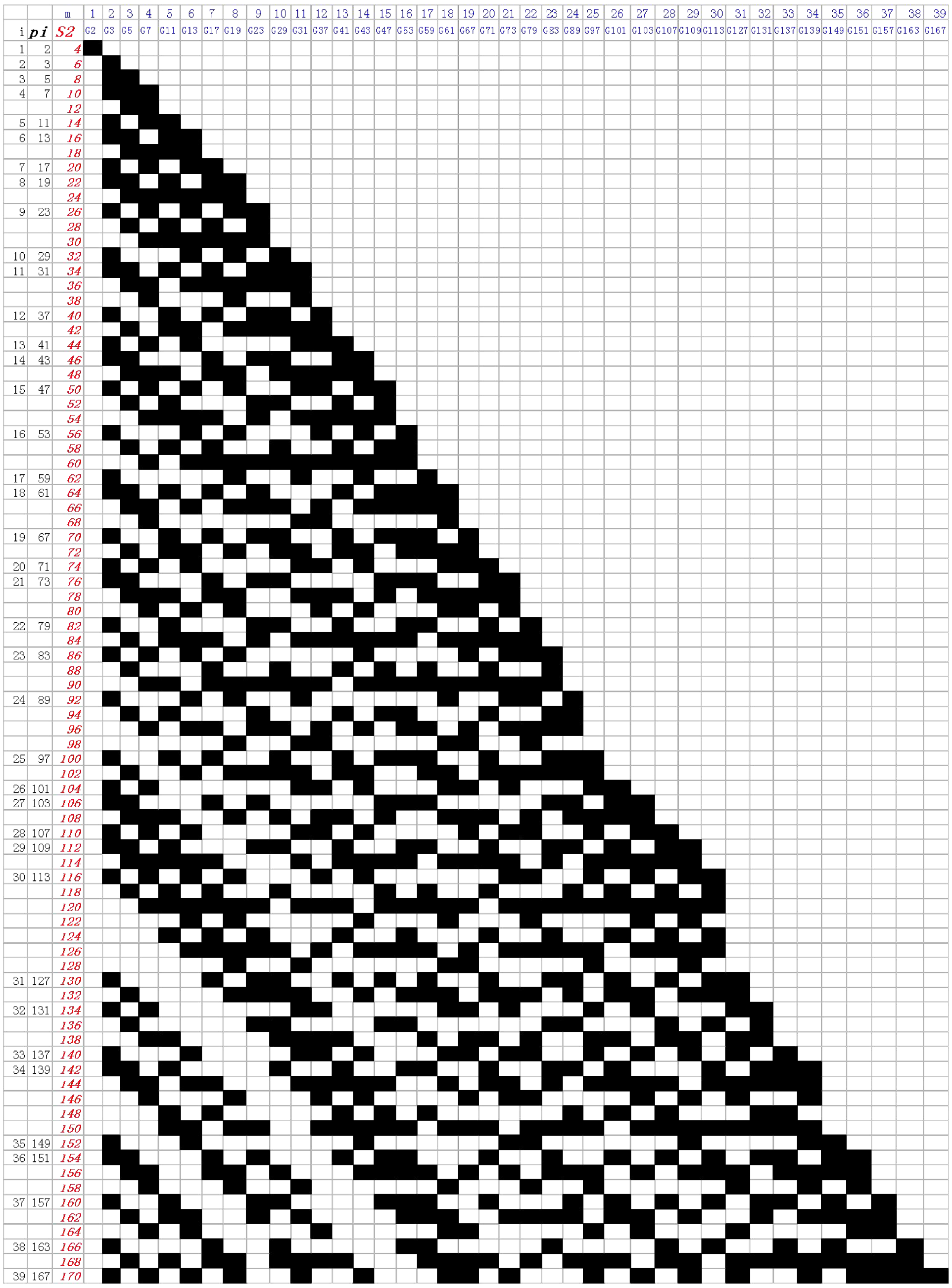

Figure 3: The equivalent prime addition table

In order to show the hole properties of the prime addition table, we can mark the even elements of the prime addition table in the even set.In order to make the table convenient, the set of even elements of the m^{th} column in the prime addition table 2 is denoted by $G_{p_m} = \{p_m + p_i; i = 1, 2, 3, ...\}$, which is a subset of H. For example, $\{p_1 + p_i; i = 1, 2, 3...\} = \{4, 5, 7, 9, 13, 15, ...\}$, where only 4 is even, so $G_{p_1} = \{4\}$. $\{p_2 + p_i; i = 1, 2, 3...\} = \{5, 6, 8, 10, 14, 16, ...\}$,the set of evens is denoted by $G_{p_2} = \{6, 8, 10, 14, 16, ...\}$. Obviously, every G_{p_m} does not contain all evens, that is, there are many even holes in G_{p_m}. Put the set $S_2 = \{2, 4, 6, 8, ...\}$ of all evens in the first column on the left, then arrange G_{p_m} in the order of primes. The elements whose G_{p_m} intersects S_2 in each column are marked (blackened) at the corresponding position of this row, the non-intersect elements are empty, and the leftmost column is the prime number corresponding to the elements in G_{p_2}. We get the equivalent prime addition table 3,which is equivalent to the primes addition table 2.

There are many vacancies in each column, which means that the sum of a prime p_m and all other primes p_i cannot form all evens. The even holes of G_{p_m} are shown in the equivalent prime addition table.The following six Lemmas are the properties of the equivalent prime addition table.

Lemma 2.1.2 *There is only one even 4 on the even axis and the other even numbers are holes in G_{p_1}.*

Proof 4 *Only 4 is even in $\{p_1 + p_i, i = 1, 2, 3, ...\}$ and every else is odd.So,*

$$G_{p_1} = \{4\}. \tag{24}$$

There is only one number 4 in the even axis and the other even numbers are holes.

Lemma 2.1.3 $G_{p_2} = \{p_i + 3; i = 2, 3, 4, ...\}$. *There are holes of G_{p_2} when $p_n - p_{n-1} \geq 4$.The numbers of holes are $k_n^{p_2} = \frac{1}{2}(p_n - p_{n-1}) - 1$.*

Proof 5 *There are no hole between the two even numbers of the difference 2.For example,there is not hole between even 6 and 8. If the difference between adjacent elements is greater than 2, the deep $k_n^{p_2}$ of hole in G_{p_2} is*

$$k_n^{p_2} = \frac{(3 + p_n) - (3 + p_{n-1})}{2} - 1 = \frac{p_n - p_{n-1}}{2} - 1 = k_n. \tag{25}$$

Compared with the definition of odd holes in prime sequence(Definition 1.3.2),the structure of even holes in G_{p_2} is the same as that of odd holes in primes sequence.

Lemma 2.1.4 *The structure of each column G_{p_m} in the equivalent addition table are same as the structure of G_{p_2}, and the mark position of G_{p_m} are displace $p_m - 3$ down relative column G_{p_2}.*

Proof 6 *For $m \geq 3$,*

$$\begin{aligned} G_{p_m} &= \{p_m + p_i; i = 2, 3, 4, ...\}, & (26)\\ &= \{p_m + 3, p_m + 5, p_m + 7, ..., p_m + p_i, p_m + p_{i+1}, ...\}. & (27) \end{aligned}$$

If

$$4 \leq (p_m + p_n) - (p_m + p_{n-1}) = p_n - p_{n-1}, \tag{28}$$

there are holes,and the hole numbers are

$$k_n^{p_m} = \frac{1}{2}((p_m + p_{n+1}) - (p_m + p_n)) - 1 = k_n. \tag{29}$$

It means that the structure of holes in G_{p_m} are same as structure of holes in G_{p_2}.The first element of G_{p_m} is $(p_m + 3)$,and the first element of G_{p_2} is 6.Therefore,the relative position of G_{p_m} to G_{p_2} same as displacement down

$$(p_m + 3) - 6 = p_m - 3. \tag{30}$$

My Figure 3 be drew by using Lemma 2.1.4.In Excel table I blackened every element of G_{p_2} first.I have done the equivalence prime addition table less than $p_{1229} = 9973$.Because the structure of all G_{p_m} is same,we copy the column of G_{p_2} to paste one by one in every first position $p_m + 3$ for the column of G_{p_m}.

Lemma 2.1.5 *In $[p_m + p_{n-1}, p_m + p_n]$,the even holes of G_{p_m} are $\{p_m + p_{n-1} + 2, p_m + p_{n-1} + 4, ..., p_m + p_{n-1} + 2k_n\}$.*

Proof 7 *According to the Lemma 2.1.3 and 2.1.4,in $[(p_m+p_{n-1}), (p_m+p_n)]$,the numbers of even holes are k_n.Therefore the position of the holes are $\{p_m+p_{n-1}+2, p_m + p_{n-1} + 4, ..., p_m + p_{n-1} + 2k_n\}$.*

Definition 2.1.6 *Family $F(p_n, m) \equiv \{p_n + 3, p_n + 5, ..., p_n + p_m\}$ =$\{p_n+p_i; i = 2, 3, 4, ..., m\}$,is a set of element $p_n + p_i$ of every column $G_{p_i}(i = 2, 3, ..., m)$ for given prime p_n.*

In Figure 3, each family of elements is an oblique column starting with $(p_n + 3)$.$F(p_2, m) = \{6, 8, 10, 14, 16, ..., p_m + 3\}$ is the set of the uppermost elements of each column in Figure 3.$F(p_{30}, m) = \{116, 118, 120, 124, ..., p_{30}+p_m\}$ is the set of elements for each column corresponding primes p_{30} in Figure 3.

Lemma 2.1.7 *The structure of every family is same.The first element of every family is in G_{p_2}.*

Proof 8 *For any p_n and* m,$\{(p_n + p_i); i = 2, 3, 4, .., m\}$ *are the elements of family $F(p_n, m)$.Because difference of adjacent elements in $F(p_n, m)$ is $(p_n + p_i) - (p_n + p_{i-1}) = p_i - p_{i-1}$,which are irrelevant to p_n.So the structure of every family $F(p_n, m)$ marked as p_n is same each other.The first element of every family is $(p_n + 3)$,it is in G_{p_2}.*

Lemma 2.1.8 *The height of the family $F(p_n, m)$ is $p_m - 3$.*

Proof 9 *For any $F(p_n, m)$,the maximum element is (p_n+p_m) and the minimum element is $(p_n + 3)$ .The height of this family is $(p_n + p_m) - (p_n + 3) = p_m - 3$.*

2.2 The Through Hole and Its Conditions

Definition 2.2.1 *The through hole.The even E is through hole of* $\bigcup_{i=1}^{m} G_{p_i}$ *if* $E \notin \bigcup_{i=1}^{m} G_{p_i}$.

For example,E=12 is through hole of $\bigcup_{i=1}^{2} G_{p_i}$.E=98 is through hole of $\bigcup_{i=1}^{7} G_{p_i}$.

Theorem 2.2.2 *The through holes in the equivalent prime addition table are in the same row.*

Proof 10 *The even hole E for* $\bigcup_{i=1}^{m} G_{p_i}$ *means that the E is hole for every set* G_{p_i}*,1*$\leq i \leq m$*,so the through hole E in same row in the equivalent prime addition table.*

Theorem 2.2.3 *The prime addition table contains all even numbers, which is equivalent to the absence of through holes in the addition table.*

Proof 11 *According to the Definition(2.0.3)*

$$H = \lim_{m \to \infty} \bigcup_{i=1}^{m} G_{p_i} = \bigcup_{i=1}^{\infty} G_{p_i}, \tag{31}$$

which is the set of all even numbers of the prime addition table.Accoding the through hole definition (Definition (2.2.1)),the existence of an even number that does not include in the prime addition table H means that there is a through hole. Conversely, the absence of through holes means that the prime addition table contains all even numbers.

Let's see the Figure 3,there is no through hole pass through the triangular region in Figure 3,but there are many through holes in unions of finite m column, $\bigcup_{i=1}^{m} G_{p_i}$.For example,E=98 is through hole of $\bigcup_{i=1}^{7} G_{p_i}$.

Theorem 2.2.4 *The through hole of triangle region is the through hole of the equivalent prime addition table.*

Proof 12 *There is two regions in* $\bigcup_{i=1}^{m} G_{p_i}$*.One is the triangle region which position* $\leq (p_m + 3)$*.Another one is the below triangle region which position* $> (p_m + 3)$*.If there are through holes, there are only two possibilities. One is in the triangle region, the other is below the triangle region. First, we study the area below the triangle.*

For finite m,there are many through holes in $\bigcup_{i=1}^{m} G_{p_i}$*,whose height of triangle region is* $(p_m + 3)$*.According to the PNT,there will always be adjacent primes whose difference is greater than the height of the triangle region,i.e,*

$$p_{N_m+1} - p_{N_m} > p_m - 3. \tag{32}$$

So we have

$$(p_{N_m+1} + 3) - (p_{N_m} + p_m) > 0. \tag{33}$$

It means that the adjacent families $F(p_{N_m}, m)$ and $F(p_{N_m+1}, m)$ are not overlap.The even between $(p_{N_m}+p_m)$ and $(p_{N_m+1}+3)$ are through holes of $\bigcup_{i=1}^{m} G_{p_i}$.For example,E=122,124,126,128 are four through holes of $\bigcup_{i=1}^{4} G_{p_i}$ in Figure 3.Clearly, such through holes are below the triangle region.

If an even number E is a hole of every adjacent K families intersecting with it, then the E is also a through hole.For example,E=98 is a through holes of families $F(p_{24}, 7)$ and $F(p_{23}, 7)$ in $\bigcup_{i=1}^{7} G_{p_i}$.Clearly, such through holes are below the triangle region.

When m increases, the above two types of through holes are not necessarily through holes.Only through holes in the triangle region, no matter how big m is, are all through holes. Therefore, the through hole of the triangle region is the through hole of the equivalent prime addition table.Therefore, only the through holes across the triangle region.

We verify first through hole when $m \leq 39$.We can directly look the position of the through hole in the equivalent prime addition table.The position E of first through hole for every $\bigcup_{i=1}^{m} G_{p_i}$ within *10000* shows in Table 3.N is the N^{th} prime of the equivalent prime addition table correspondence with E.

Table 3 The position of the first through hole of the m columns set

m	E	N		m	E	N		m	E	N
1	6	2		14	556	101		27	5372	708
2	12	4		15	992	166		28	5372	708
3	30	9		16	992	166		29	5372	708
4	98	24		17	992	166		30	5372	708
5	98	24		18	992	166		31	5372	708
6	98	24		19	992	166		32	5372	708
7	98	24		20	992	166		33	5372	708
8	220	47		21	2642	382		34	7426	941
9	308	62		22	2642	382		35	7426	941
10	308	62		23	2642	382		36	7426	941
11	556	101		24	2642	382		37	7426	941
12	556	101		25	2642	382		38	7426	941
13	556	101		26	2642	382		39	7426	941

For example, $m = 7$.It means that the depth of the hole is *7* columns, but the hole position is $E_7 = 98$, the correspondence column number in the equivalent prime addition table is 24,and these columns that can be used to block the hole are $N_7 = 24 > 7$.$E = 98$ is the through hole for $\bigcup_{i=1}^{4} G_{p_i}, \bigcup_{i=1}^{5} G_{p_i}, \bigcup_{i=1}^{6} G_{p_i}, \bigcup_{i=1}^{7} G_{p_i}$.

In 2013,Tomas et al have verified Goldbach's conjecture hold within 4×10^{18} [2].Corresponding $m \approx 1.0 \times 10^{17}$.Therefore,there is not through hole in the triangle in $\bigcup_{i=1}^{m} G_{p_i}$ for $m \leq 1.0 \times 10^{17}$.Therefore, the position of the first through hole is greater than 4×10^{18}.

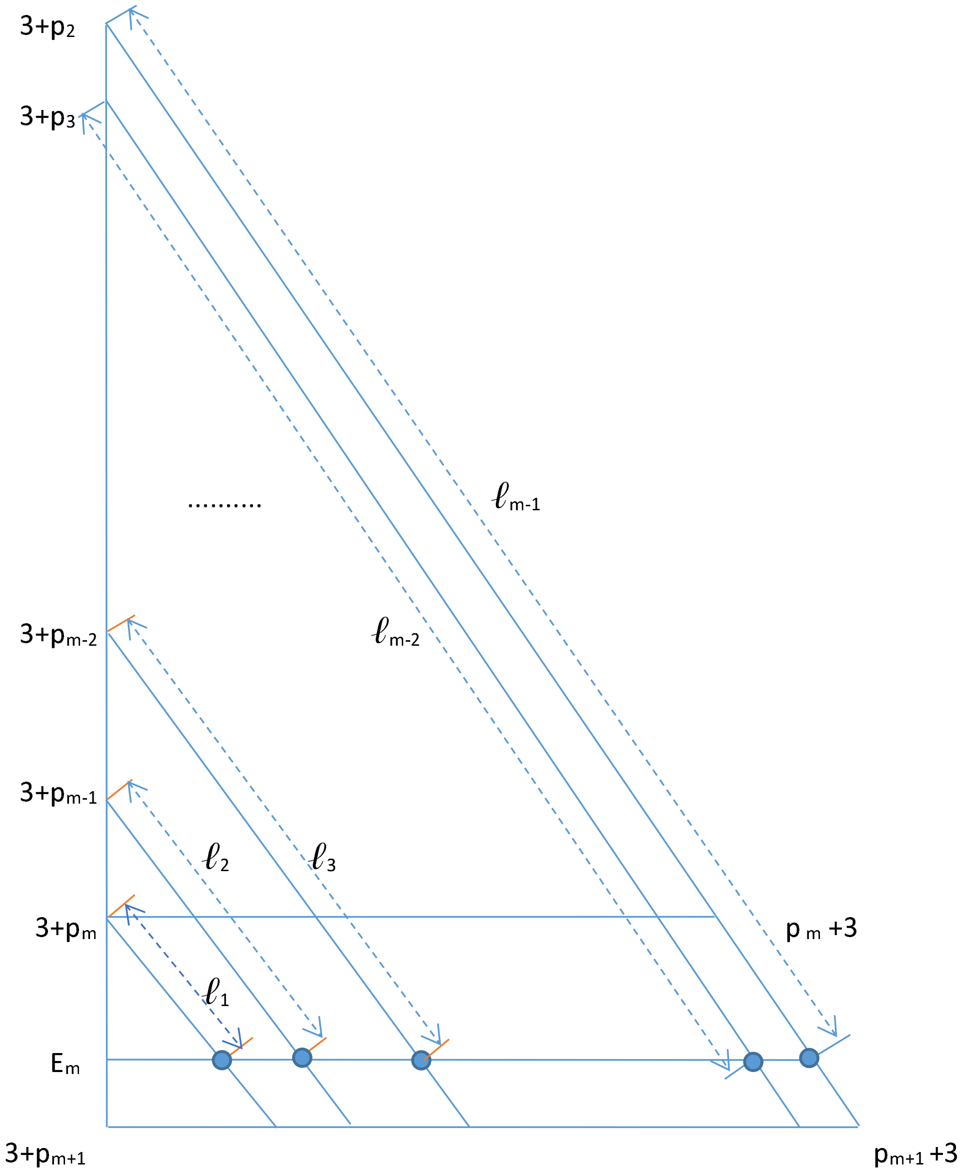

Figure 4: The through hole in triangle

Theorem 2.2.5 *The forming condition of through hole is that the negative prime sequence and its corresponding non-prime odd sequence have complemen-*

tary symmetry of translation E.

Proof 13 *According to the Theorem (2.2.2),The forming condition of through holes is that all prime families intersecting the through holes have a common hole.Without losing generality,assume that there is not through hole before* (p_m+3)*,and there is one through hole E in* $[p_m+3, p_{m+1}+3]$ *in the triangle region,so* $(3+p_m) < E < (3+p_{m+1})$ *(See Figure 4).*

Therefore the E is the even hole for every one of the $(m-1)$ *families,and the* $(m-1)$ *families must be satisfied following equations,*

$$E = 3+p_m+l_1 = 3+p_{m-1}+l_2 = 3+p_{m-2}+l_3 = ... = 3+p_3+l_{m-2} = 3+p_2+l_{m-1}, \tag{34}$$

where l_i *is distance between even holes from* (p_i+3) *to* (p_i+b_i) *in family* $F(p_i, m)$*.It satisfied that*

$$l_i = (p_i+b_i) - (p_i+3) = b_i - 3. \tag{35}$$

where b_i *is non-prime odds,*$b_i \in O_{P_m}$ *in family* $F(p_{m-i}, m)$*.According to the Formula (19),the* b_i *is some of the* o_l*.So,the Formula(34) becomes*

$$E = p_m + b_1 = p_{m-1} + b_2 = p_{m-2} + b_3 = ... = p_3 + b_{m-2} = p_2 + b_{m-1}. \tag{36}$$

It can be written as

$$\begin{cases} b_1 = (-p_m) + E \\ b_2 = (-p_{m-1}) + E \\ ... \\ b_{m-2} = (-p_3) + E \\ b_{m-1} = (-p_2) + E \end{cases} \tag{37}$$

It means that the negative primes sequence $\{-p_m, -p_{m-1}, ..., -p_3, -p_2\}$ *and the non-prime odds sequence* $\{b_1, b_2, ..., b_{m-2}, b_{m-1}\}$ *are symmetry of translation E.We will study translation symmetries of number sequences next section.*

3 The Symmetry of Prime Sequence

There is its internal structure for any number sequence.Therefore there are some symmetries for any number sequence.Many laws can be obtained by studying the symmetries of these number sequence.The translation symmetry contains the law of the sum of two numbers.The prime sequence is a special one, and its translation asymmetry implies Goldbach's conjecture. In order to study the symmetry of prime sequence, we study first the symmetry of natural sequence, even number sequence and odd number sequence.

3.1 Symmetry of Natural Sequence

Natural number sequence is $N = \{i; i = 0, 1, 2, 3, ...\}$. N can be expressed intuitively in the one-dimensional X-axis, that is, all integer points on the X-axis from 0 every one.

Definition 3.1.1 *Inversion of number sequence,σ.Let $S = \{s_i; i = 1, 2, 3, ...\}$,then $\sigma S = \{-s_i; i = ..., 3, 2, 1\}$,which is inversion of number sequence S with 0 as the origin.The arrangement of number sequence is from small to large.*

The negative nature number sequence of N is $N^- = \sigma N = \{-i; i = 0, 1, 2, 3, ...\} = \{..., -3, -2, -1, 0\}$. The union of N and N^- is an integer sequence,$Z = \{N, N^-\} = \{-i, i; i = 0, 1, 2, 3, ...\}$. The density of Z is constant to 1, that is, the integers are uniformly distributed.

Definition 3.1.2 *Translation of number sequence,T_m.Given a number sequence $S = \{s_i; i = 0, 1, 2, 3, ...\}$ and nature number m.Translation m of sequence S means that each number s_i plus m,denoted by T_m.So,*

$$S' = T_m S = \{s_i + m; i = 0, 1, 2, 3, ...\}. \tag{38}$$

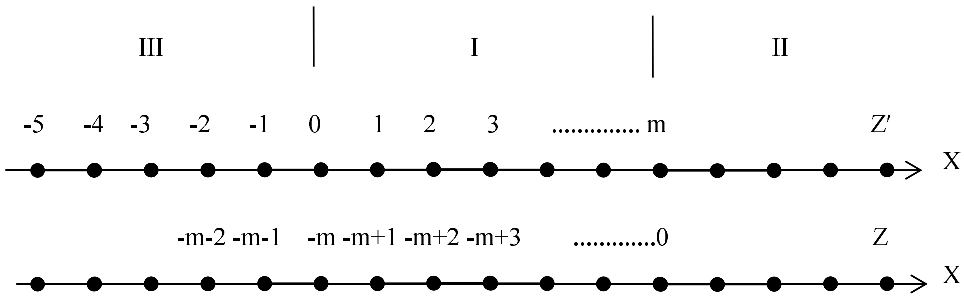

Figure 5: The translation of nature number sequence

For example, if the integer sequence Z translates the integer m, another sequence Z' is obtained.See Figure 5.

$$\begin{aligned} Z' &= T_m Z = \{-i + m, i + m; i = 0, 1, 2, 3, ...\}, & (39)\\ &= \{..., -3 + m, -2 + m, -1 + m, m, 1 + m, 2 + m, 3 + m, ...\}. & (40)\end{aligned}$$

Definition 3.1.3 *Translation symmetry. Suppose S' is a sequence of S after translation m.If sequence S and S' are the same structure,same distribution and indistinguishability. We call that S has symmetry with translation nature number m.*

Theorem 3.1.4 *Any natural number m can be expressed as the sum or difference of two natural numbers.*

Proof 14 *Given integer sequence* Z*,*$Z' = T_m Z$*. The* Z *and* Z' *intersect an every point and are indistinguishable.So,*Z *is symmetrical with translation* m*. Discuss in three zones I, II and III(See Figure 5).*

(1) Zone I. The Z' *sequence is that the* $\{0, 1, 2, 3, ..., m\}$ *in X-axis positive direction.The* Z *sequence is* $\{-m, ..., -3, -2, -1, 0\}$ *in X-axis negative direction. let* $z' \in Z'$*,*$z \in Z$*,we have* $0 < z' < m$*,*$-m < z < 0$*,*

$$z' = z + m. \tag{41}$$

Let n_1 *and* n_2 *are natural numbers corresponding to integer sequence* Z *and* Z'*.So* $z' = n_1$ *and* $z = -n_2$*, i.e.*

$$m = z' - z = n_1 + n_2. \tag{42}$$

It means that every nature number m *can be expressed as the sum of two natural numbers* n_1 *and* n_2*.*

Here, the intersection point are at $x = 0, 1, 2, ..., m$*, which is expressed as the sum of two natural numbers in* $(m + 1)$ *ways, namely*

$$m = 0 + m = 1 + (m - 1) = 2 + (m - 2) = ... = m + 0. \tag{43}$$

(2) Zone II. $z' > m$*,*$z > 0$*,* $z' = z + m$*. Because* $z' = n_1$ *and* $z = n_2$*, that is,* $m = n_1 - n_2$*, the nature number* m *can be expressed as the difference between two natural numbers* n_1 *and* n_2*.For a fixed* m*,there is infinite intersection points,and then there is infinite combinations.*

(3) Zone III.$z' \leq 0$*,*$z \leq -m$*,* $z' = z + m$*. Because* $z' = -n_1$ *and* $z = -n_2$*, that is,* $m = n_2 - n_1$*, the nature number* m *can be expressed as the difference between two natural numbers* n_2 *and* n_1*.For a fixed* m*, there is infinite combinations.*

Above three cases are combined,that is, any natural number m *can be expressed as the sum or difference of two natural numbers.*

In particular, it is pointed out that in region I, the positive Z_m *and negative* Z_m^- *coincide exactly with each other. As a result,every nature number* m *can be expressed as the sum of two natural numbers.*

3.2 Symmetry of Even and Odd Sequences

The even number sequence $E^+ = \{0, 2, 4, 8, ...\}$ is all integer points on the X-axis starting from 0 every two. We got the even negative sequence,$E^- = \sigma E^+ = \{..., -8, -6, -4, -2, 0\}$. The union of E^+ and E^- is $\ddot{E} = \{E^+, E^-\}$,a full even sequence. The full even number sequence density is $\frac{1}{2}$, uniformly distributed.

The odd number sequence $O^+ = \{1, 3, 5, 7, 9, ...\}$ is all the integer points on the X-axis starting from 1 every two.The odd negative sequence,$O^- = \sigma O^+ = \{..., -9, -7, -5, -3, -1\}$. The union of O^+ and O^- is $\ddot{O} = \{O^+, O^-\}$,a full odd sequence. The full odd number sequence density is $\frac{1}{2}$, uniformly distributed.

Obviously, a proof identical to Theorem 3.1.4,the full even number sequence $\ddot{E}$ and the full odd number sequence $\ddot{O}$ have translating symmetry even number respectively, so there are three lemmas as follows:

Lemma 3.2.1 *Even number sequence $\ddot{E}$ are symmetry with translating any even number E, then any even number E can be expressed as the sum or difference of two even numbers e_1 and e_2. That is:(i) in zone I,$E = e_1 + e_2$;(ii) in zone II and III, $E = e_1 - e_2, (e_1 > e_2)$ or $E = e_2 - e_1, (e_2 > e_1)$.*

Lemma 3.2.2 *Odd number sequence $\ddot{O}$ are symmetry with translates any even number E, so any even number E can be expressed as the sum or difference of two odd numbers o_1 and o_2. That is:(i) in zone I,$E = o_1 + o_2$; (ii) in zone II and III, $E = o_1 - o_2, (o_1 > o_2)$ or $E = o_2 - o_1, (o_2 > o_1)$.*

Definition 3.2.3 *Translation complementary symmetry. Given two non-intersection sequence S_1 and S_2,i.e,$S_1 \bigcap S_2 = \emptyset$. If S_1 is completely coincident with S_2 after translating m, the sequence S_1 and S_2 have the complementary symmetry of translating m.*

Lemma 3.2.4 *The even sequence $\ddot{E}$ and odd sequence $\ddot{O}$ has complementary symmetry with the translation every odd number O. Then any odd number O can be expressed as the sum or difference between an even number e and an odd number o. That is:(i) in zone I,$O = o+e$; (ii) in zone II and III, $O = e-o, (e > 0)$ or $O = 0 - e, (o > e)$.*

Here, O is expressed as the sum of even and odd numbers in $\frac{O+1}{2}$ ways, namely

$$O = 0 + O = 2 + (O - 2) = 4 + (O - 4) = ... = (O - 1) + 1. \tag{44}$$

3.3 Symmetry of Prime Sequences

In the first section, we obtain a prime sequence $P = \{p_i; i = 1, 2, 3, ...\}$ whose distribution satisfies the Prime Number Theorem.A piece of prime sequence denoted by $P_m = \{p_i; i = 1, 2, 3, ..., m\}$. Since $P_1 = \{2\}$ is even, the sum or difference between it and any other primes is odd number.When discuss translational even number, we delete P_1 and get the prime sequence $P^o = P - \{p_1\} = \{p_i; i = 2, 3, 4, ...\} = \{3, 5, 7, 11, ...\}$,which is all odd prime sequence.Without causing confusion,we omit the superscript o of P^o and mark it as P.

According to the Definition 1.3.2,there are $k_n = \frac{1}{2}(p_n - p_{n-1}) - 1$ non-prime odds holes between p_n and p_{n-1}.The negative sequence $P^- = \sigma P^+ = \{-p_i; i = 2, 3, 4, 5, ...\} = \{..., -11, -7, -5, -3\}$ is obtained. The union set of P^+ and P^- is the full odd prime sequence $P = \{P^+, P^-\}$. In this way, the density distribution of P in both positive and negative directions of the X-axis are same and satisfies the Prime Number Theorem ,which is not uniform.

Definition 3.3.1 *The odd complement set O_P of the odd prime sequence P is $O_P = O - P$. That is the all odd numbers except the odd prime sequence,*

$$O_P = \{..., -25, -21, -15, -9, -1, 1, 9, 15, 21, 25, ...\}. \tag{45}$$

Lemma 3.3.2 *The odd prime sequence P translates any odd number O into even number sequence $\ddot{E}$. That is, any odd number O can be expressed as the sum or difference of odd prime p_n and even number e.That is:(i) in zone I,$O = p_n + e$;(ii) in zone II and III, $O = p_n - e, (p_n > e)$ or $O = e - p_n, (e > p_n)$.*

The proof of above Lemma identical to the Theorem 3.1.3.

Lemma 3.3.3 *The odd prime sequence P is asymmetry of translation even E.*

Proof 15 *According to the Prime Number Theorem, the distribution of P is not uniform.By translating even number E,*

$$P' = T_E P = \{p_i + E; i = 2, 3, 4, ...\}. \tag{46}$$

The P' does not coincide with P at every prime, so the prime sequence is not symmetric with translation even number E.

In zone I,a prime sequence is $P_m = \{p_i, i = 2, 3, 4, ..., m; P_m < E\}$.The reversion of P_m is

$$P_m^- = \{-p_i, i = 2, 3, 4, ..., m; P_m < E\} = \{-p_m, ..., -5, -3\}. \tag{47}$$

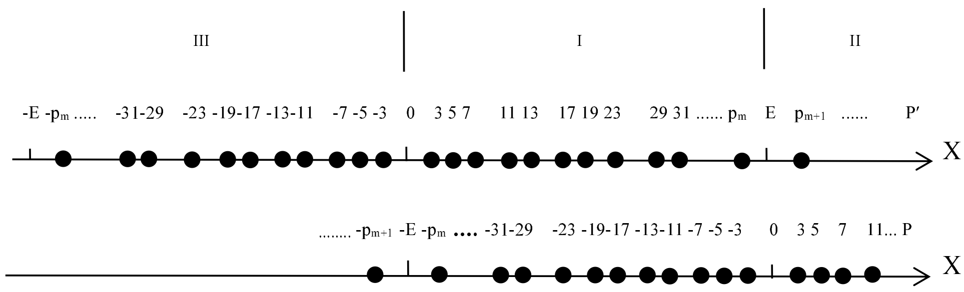

Figure 6: The translation of prime sequence

Obviously,after translating even number E,P_m and P_m^- are not intersection at every prime,but they may be intersection at some point.

Theorem 3.3.4 *If the P_m and $T_E P_m^-$ are intersection at some primes(at least one prime),the even number E can be expressed as the sum of pairs of these primes,at least one pair of primes.*

Proof 16 *If the P_m and $T_E P_m^-$ are intersection at some primes(at least one prime),we can find some prime $p_i \in P_m$ and $-p_j \in P_m^-$,which satisfy $p_i = (-p_j) + E$.Therefore,$E = p_i + p_j$.*

Theorem 3.3.5 *If the P_m and $T_E P_m^-$ are not intersection at every prime,this even number E can not be expressed as the sum of prime pair.Such even number E can be expressed as the sum of every prime less than E and their non-prime odd.*

Proof 17 *That is to say, after translating even number E,there is not intersection prime of odd prime sequence* $P_m = \{3, 5, ..., p_m\}$ *and* $P_m^- = \{-p_m, ..., -5, -3\}$. *Therefore,*$P_m^-$ *fall into its odd complement set* O_{P_m},

$$o_i = (-p_i) + E. \tag{48}$$

This shows that these even numbers E are the sum of prime $p_i \in P_m$ *and non-prime odd numbers* $o_i \in O_{P_m}$.*That is,*$E = p_i + o_i$. *Obviously,*$i = 1, 2, 3, ..., m$.*So,the even number E cannot be expressed as the sum of two prime numbers. Such an even number E is the Goldbach's exception number.*

If There are many such Goldbach's exception numbers,which denoted by $E_m, m = 1, 2, 3, ...$.The finite m means that there are finite number of Goldbach's exception number.The infinite m means that there are infinite number of Goldbach's exception number.Through actual verification, until 4×10^{18} [2], no Goldbach's exception number has been found, so $E_1 > 4 \times 10^{18}$.

Theorem 3.3.6 *The m is finite.Namely,the number of Goldbach's exceptions is finite.*

Proof 18 *Suppose there are* E_m, *m = 1,2,3,... .The m can be infinite.For every* E_m, *we have an odd prime sequence,*

$$P_{n_m} = \{3, 5, ..., p_{n_m}\}, \tag{49}$$

where n_m *satisfies*

$$p_{n_m} < E_m, \tag{50}$$

p_{n_m} *is the first prime less than* E_m.*Therefore,we can get correspondence negative sequence* $P_{n_m}^-$,

$$P_{n_m}^- = \{-p_{n_m}, -p_{n_m-1}, ..., -5, -3\}. \tag{51}$$

Corresponding the sequence P_{n_m},*we have its odd complement set,*

$$O_{P_{n_m}} = O - P_{n_m} \tag{52}$$
$$= \{o_i; i = 0, 1, 2, ..., K_{n_m}\} \tag{53}$$
$$= \{1, 9, 15, ..., o_{K_{n_m}-1}, o_{K_{n_m}}\}, \tag{54}$$

where

$$K_{n_m} = \frac{1}{2}(p_{n_m} - 3) - (n_m - 1). \tag{55}$$

In zone I, If E_m *is through hole,after translating* E_m, *the sequence* $P_{n_m}^-$ *falls in the* $O_{P_{n_m}}$. *The* $(n_m - 1)$ *odd primes fall in* $(n_m - 1)$ *positions of* K_{n_m} *elements of* $O_{P_{n_m}}$. *The most possible filling methods of* K_{n_m} *elements of* $O_{P_{n_m}}$ *are* $K_{n_m}!$,*which is finite.*

For example, suppose that $E_1 \sim 4 \times 10^{18}$, *according to the Prime Number Theorem,*

$$p_{n_1} \approx n_1(\ln n_1 + \ln\ln n_1 - 1) \approx 4 \times 10^{18}, \tag{56}$$

we got

$$n_1 \approx 9.6 \times 10^{16}. \tag{57}$$

There is a prime number sequence

$$P_{9.6\times10^{16}} = \{3, 5, ..., p_{9.6\times10^{16}}\}, \tag{58}$$
$$P^-_{9.6\times10^{16}} = \{-p_{9.6\times10^{16}}, ..., -5, -3\}. \tag{59}$$

Which means that P_{n_1} *have* $(n_1 - 1)$ *elements.Therefore,there is correspondence odd sequence*

$$O_{P_{9.6\times10^{16}}} = \{1, 9, 15, ..., o_{K_{9.6\times10^{16}-1}}, o_{K_{9.6\times10^{16}}}\}, \tag{60}$$

where

$$K_{9.6\times10^{16}} \approx 1.9 \times 10^{18}. \tag{61}$$

The prime numbers only account for non-prime odd numbers

$$\frac{n_1}{K_{n_1}} \approx 5\%. \tag{62}$$

By translating even E_1 *of the prime sequence* $P^-_{n_1}$ *,the* $(9.6 \times 10^{16} - 1)$ *prime numbers of* $P^-_{n_1}$ *will fall into this* $K_{n_1} \approx 1.9 \times 10^{18}$ *non-prime negative odd number position of* $O_{P_{n_1}}$*,but it must be one of the* $K_{n_1}!$ *kinds. See Figure 6 (a).*

For E_2*, there exists a primes sequence*

$$P_{n_2} = \{3, 5, ..., p_{n_1}, p_{n_1+1}, ..., p_{n_2}\}, \tag{63}$$
$$P^-_{n_2} = \{-p_{n_2}, ..., -p_{l(n_1,n_2)}, -p_{l(n_1,n_2)-1}, ..., -5, -3\}. \tag{64}$$

where $l(n_1, n_2)$ *satisfies*

$$p_{n_2} - p_{l(n_1,n_2)} \approx p_{n_1} - 3, \tag{65}$$

means the length of prime sequence $\{p_{l(n_1,n_2)}, ..., p_{n_2}\}$ *is approximately equal to the length of prime sequence* $\{3, 5, ..., p_{n_1}\}$*.There is correspondence non-prime odd numbers set of* $O_{P_{n_2}}$*,*

$$O_{P_{n_2}} = \{1, 9, 15, ..., o_{K_{n_2}}\}, \tag{66}$$
$$= O_{P_{n_1}} + \{o_{K_{n_1}+1}, ..., o_{K_{n_2}}\}. \tag{67}$$

The prime number sequence $\{-p_{n_2}, ..., -p_{l(n_1,n_2)}\}$ *should also be filled in* $O_{P_{n_1}}$*.The filling method is not necessarily same with the method in* E_1*, but it must be one of their* $K_{n_1}!$ *kinds. See Figure 6 (b).*

By analogy, for E_m*, there is a prime sequence*

$$P_{n_m} = \{3, 5, ..., p_{n_1}, p_{n_1+1}, ..., p_{n_m}\}, \tag{68}$$
$$P^-_{n_m} = \{-p_{n_m}, ..., -p_{l(n_1,n_m)}, -p_{l(n_1,n_m)-1}, ..., -5, -3\}, \tag{69}$$

where $l(n_1, n_m)$ *satisfies*

$$p_{n_m} - p_{l(n_1,n_m)} \approx p_{n_1} - 3. \tag{70}$$

Therefore there is a non-prime odd numbers sequence

$$O_{P_{n_m}} = \{1, 9, 15, ..., o_{K_{n_m}}\}, \tag{71}$$
$$= O_{P_{n_1}} + \{o_{K_{n_1}+1}, ..., o_{K_{n_m}}\}. \tag{72}$$

Translation E_m *of* $P^-_{n_m}$*,*$P^-_{n_m}$ *falls into the set of non-prime odd numbers* $O_{P_{n_m}}$ *.It is important to point that this prime* P^-_{nm} *must also be filled in* $O_{P_{n_1}}$*.The filling method is not necessarily the same* p_{n_1}*, but it must be one of their* $K_{n_1}!$ *kinds. See Figure 6 (c).*

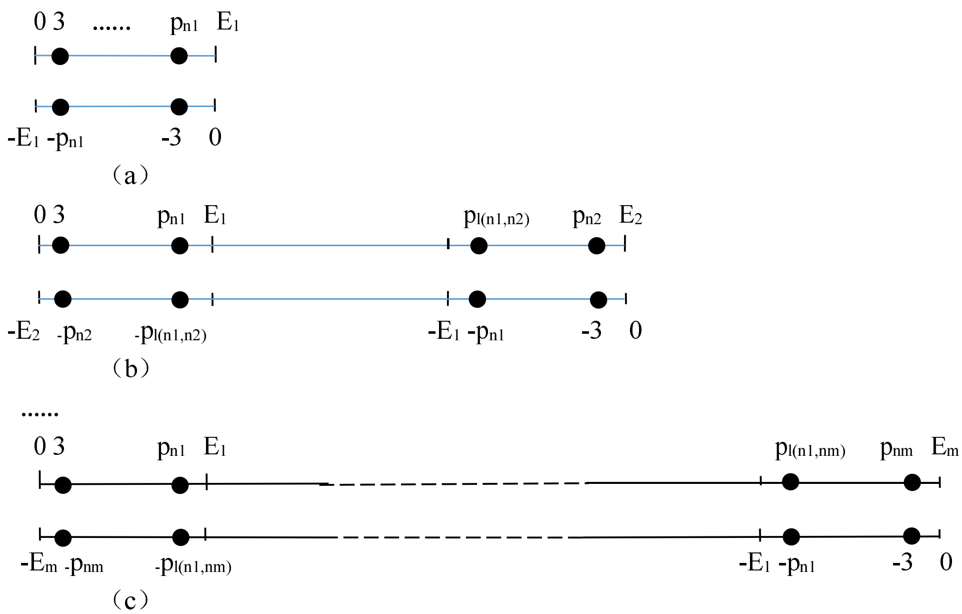

Figure 7: The intersection of prime sequence

If m *is enough large,there are many* m *which satisfies*

$$m > K_{n_1}!. \tag{73}$$

It means that there are always some kind of filling that would repeat, indicating that the distribution of the corresponding segment primes $\{-p_{n_m}, ..., -p_{l(n_1,n_m)}\}$ *would repeat.*

If m *are infinite, some kind of prime distribution recur infinitely. This contradicts the PNT.*

Notice that the first Goldbach's exception E_1 *for our discussion above may be any* E_i*,*$(1 < i \leq m)$*. It means that every following prime sequence* P_{n_m} *must* *fill into before every non-prime odd sequence,*$O_{P_{n_1}}$,$O_{P_{n_2}}$,...,$O_{P_{n_{m-1}}}$*.Therefore, not only for segment primes* $\{-p_{n_2}, ..., -p_{l(n_1,n_2)}\}$*, but also for segment primes* $\{-p_{n_m}, ..., -p_{l(n_1,n_m)}\}$*. This means that any distribution of a prime number can be repeated infinitely, which violates the Prime Number Theorem. Therefore,* m *is finite and the number of Goldbach's exceptions is finite.*

From Theorem 3.3.6, the number of Goldbach exceptions is finite, denoted by M, and the corresponding even number is denoted by E_M. So there are the following corollaries:

Corollary 3.3.7 *Any even number* E *satisfying* $2 < E < E_1$ *and* $E > E_M$*, can be expressed as the sum of two primes.*

Therefore,there is a special interval (E_1, E_M).I prove now that there is not such interval (E_1, E_M) according to the PNT.

Theorem 3.3.8 *There is not such interval* (E_1, E_M)*.*

Proof 19 *The existence of through holes in each interval must be due to the special distribution of primes in this interval.If only one even interval has through holes, then the distribution of primes in the interval* (E_1, E_M) *is different to the distribution of all prime sequence.Therefore, the existence of special interval* (E_1, E_M) *is inconsistent with the approximate continuity of prime distribution(Corollary 1.2.2).*

Further, because primes are infinite, no matter how big M is,

$$\lim_{n\to\infty} \frac{E_M - E_1}{p_n} \to 0, \tag{74}$$

the interval (E_1, E_M) *is very small interval,and just like a singular point in starting point for large* n*.It is contradict with the distribution theorem PNT(Corollary 1.2.2).Therefore,there is not such interval* (E_1, E_M)*.*

According to above Theorem,we have following theorem.

Theorem 3.3.9 *The prime addition table contains all even numbers greater than 2.*

Proof 20 *According to the Theorem 3.3.6 and 3.3.8,the PNT negates the existence of Goldbach's exception* E*.According to the Theorem 2.2.5,there is not even number* E *means that the* P_m^- *and* O_{P_m} *are asymmetrical.So the Equation (36) is not valid.Therefore there is not even through hole* E *in the triangle of* $\bigcup_{i=1}^{m} G_{p_i}$*.According to Theorem 2.2.3,there is not even through hole in the triangle region means that there is not even through hole in the equivalent prime addition table.It is that the prime addition table contains all even numbers greater than 2.*

References

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